NPTEL Basic Linear Algebra 2020
Assignment 5 - Subjective

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Deadline: Wednesday, April 1, 2020, 23:59 IST

(1) Solve the following equation for x.

$$\det \begin{bmatrix} 3 & -4 & 7 & 0 & 6 & -2 \\ 2 & 0 & 1 & 8 & 0 & 0 \\ 3 & 4 & -8 & 3 & 1 & 2 \\ 27 & 6 & 5 & 0 & 0 & 3 \\ 3 & x & 0 & 2 & 1 & -1 \\ 1 & 0 & -1 & 3 & 4 & 0 \end{bmatrix} = 0.$$ **[6]**

(2) Using determinants, check whether the following matrix A is invertible, and if yes, find its inverse.

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 4 \\ 1 & 2 & 3 \end{bmatrix}.$$ **[4]**

(3) Find the polynomial $A(t)$ and $p \in \mathbb{Z}^+$ such that

$$\det \begin{bmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t & t^2 \\ t^2 & t & 1 & t \\ t^3 & t^2 & t & 1 \end{bmatrix} = (1 - A(t))^p.$$ **[4]**

(4) Assume that all the matrices mentioned are of appropriate sizes.

(a) Let X, Y be matrices. Show that

$$\det \begin{bmatrix} X & Y \\ O & I \end{bmatrix} = \det X.$$ **[2]**

(b) Let A, B be matrices. Show that

$$\det \begin{bmatrix} O & A \\ -B & I \end{bmatrix} = \det AB.$$ **[2]**

(c) Prove or disprove. For two matrices A, B,

$$\det \begin{bmatrix} A & B \\ B & A \end{bmatrix} = \det(A+B)\det(A-B).$$ **[2]**

(5) Let

$$\mathcal{P}_4 = \{f(X) = a_0 + a_1X + a_2X^2 + a_3X^3 + a_4X^4 : a_0, a_1, a_2, a_3, a_4 \in \mathbb{R}\}.$$

Define $D : \mathcal{P}_4 \to \mathcal{P}_4$ as

$$D(f(X)) = f''(X), \quad \text{for all } f(X) \in \mathcal{P}_4.$$

Show that $\mathcal{P}_4$ is a vector space, D is a linear transformation, and find $\ker D$. **[5]**