## NPTEL INDUSTRIAL AND MANAGEMENT ENGINEERING DEPARTMENT, IIT KANPUR QUANTITATIVE FINANCE MID-TERM EXAMINATION (2015 JULY-AUG ONLINE COURSE)

## READ THE INSTRUCTIONS VERY CAREFULLY

1) There are Four questions and you are required to answer all of them.
2) Question Paper Uploaded on $\mathbf{2 2}^{\text {nd }}$ August, 2015, 00:00 Hrs.
3) Deadline for submission is Sunday; $\mathbf{2 3}^{\text {rd }}$ August, 2015 at 23:59 Hrs
4) This is an examination so no request for extension of deadlines will be entertained.
5) The total marks is $\mathbf{5 0}$.
6) To get full credit do your calculations carefully.
7) Submit all your written answers in a single PDF file. Excel files can be zipped together with your answer scripts combined as PDF and submitted as a single file. Kindly mention your name on all pages of your answer scripts.

## Question \# 1: [Marks\#15 (8+7)]

(a) You have an initial wealth of Rs. $W_{0}$ and it has a probability, $p$, to suffer a loss that will force you to lose half of your wealth $\left(\mathrm{L}=\frac{1}{2} W_{0}\right)$; else you will have all the wealth left with you. Your friend, Mr. Dhanilal Bimawala, an MBA form IIT Kanpur and founder of NPTEL Insurance Agency, advised you to insure against this loss. He tells you about a simple insurance policy of his company which is as follows:
If you buy Rs. $Y$ of insurance coverage policy, you have to pay $R s$. $(q Y)$ right now, and will get Rs. $Y$ from the insurance company if you suffer the disaster and you will get nothing if you do not suffer any disaster. Here $q$ the premium per dollar of insurance.
(i) Mr. Dhanilal's company is a risk-neutral company that operates in a competitive insurance market of India. If a claim for Rs. $Y$ arises, his company incur an administrative cost of $R s$. (cY) to investigate and process it. Find the expected profit of NPTEL Insurance Agency on a contract for Rs. $Y$ of insurance coverage. It is further noticed that due to competition in the market, Mr. Dhanilal is having zero expected profit on each such contract mentioned above, what relation must link $q, p$, and $c$ ?
(ii) Mr. Dhanilal found out that you have a utility-of-consequences function with a constant coefficient of relative risk aversion, $r$. He asks his financial advisor, Mr. Ganit Vittbhat, to find out the expression for your expected utility when you buy Rs. $Y$ of insurance coverage. What is the expression that Mr. Ganit Vittbhat is going to come up with for your expected utility function?
(iii) As you are also enrolled for a course on Quantitative Finance, you are expected to maximizing this expected utility with respect to $Y$. Find a formula for the fraction $Y /(0.5$ $W_{0}$ ) of your loss that you will choose to cover, as a function of $q, p$, and $r$.
(iv) Using your formula obtained above, taking $\mathrm{p}=0.1$, two cases of $c$, namely $c=0.1$ and $c=0.2$, and three cases of $r$, namely $r=0.25, r=1$, and $r=10$ (six calculations in all). In each case, the price of insurance $q$ is to be set at its competitive equilibrium level. Explain your finding for the case $r=0.25, c=0.2$.
(b) After securing your wealth, Mr. Dhanilal Bimawala gave you an investment opportunity to increase your property by investing in a mutual fund. A mutual fund of risky assets, $\mathbf{M}$, has an expected return of $16 \%\left(R_{m}=16 \%\right)$ per period and a standard deviation of $20 \%\left(\sigma_{m}=20 \%\right)$; the risk free asset, F has a guaranteed return of $8 \%\left(R_{f}=8 \%\right)$ per period.
Mr. Dhanilal tells you about the characteristics of the alternative portfolios and gives you a day time to think on your investment alternatives. You want to check different investment alternatives and invest judiciously. Answer the following questions below which will help you to find out the best investment alternative:
(i) What is the expected return and standard deviation of a portfolio that is totally invested in the risk free asset?
(ii) What is the expected return and standard deviation of a portfolio that has $50 \%$ of its wealth in the risk free asset and $50 \%$ in M ?
(iii) What is the expected return and standard deviation of a portfolio that has $100 \%$ of its wealth in M ?
(iv) What is the expected return and standard deviation of a portfolio that has $200 \%$ of its wealth in M , financed by borrowing $100 \%$ of its wealth at the risk free rate?
(v) Suppose you began the period with a net worth (wealth) of $\$ 1,000$. What is your end of period net worth in part (c) above compared with part (d), assuming the risky mutual fund, M, earns its expected rate of return. For both (c) and (d), calculate the range in your end-of-period net worth, in dollars, if the return on M is one standard deviation higher than expected (. 36 rather than .16) versus one standard deviation lower than expected (-. 04 rather than .16).
(vi) Is leverage good or bad? Have you ever put any money into a financial institution with leverage much greater than $100 \%$ ?

## Question \# 2: [Marks\#15]

After successfully completing your course in Quantitative Finance, you have been hired by an investment firm NPTEL Asset Management Pvt Limited. Mr. Rupiyachand Treasuriwala, a portfolio manager at the same firm, gives you Rs. 1 Lakh to invest. Your analysts have provided you with a list of recommended stocks: Microsoft (A), GE (B) and Exxon (C). Your job is to figure out how to invest the money in these stocks.
This question has to be done in an EXCEL (*.xlsx/*xls) file. Use the latest data for each stock and the corresponding period's S\&P 500 index to proxy for the market. Here are the instructions on how to get the necessary data.

1. Go to Yahoo Finance and download the daily prices from January 2014 to March 2015.
2. Using the historical prices, compute the daily returns for these stocks (daily data gives us better estimates of variance and covariance than monthly or yearly data).
(i) Calculate the sample mean and sample standard deviation of the historical daily returns for the three companies. Use RET column which means return including dividends. You may find the excel spreadsheet functions average and stdev useful for this. Also calculate the sample correlation matrix for the monthly returns of the three companies. You may find the Excel spreadsheet function correl useful for this. For this problem, you must express the sample means, standard deviations, and correlations accurate to 5 decimal places.
(ii) Use the sample means, standard deviations, and correlation matrix that you calculated in part (i) to calculate the sample mean and sample standard deviation of a portfolio that is (I) equally weighted in each of the three stocks, and (II) weighted according to the ratio $3: 2: 1 \mathrm{in} \mathrm{A}, \mathrm{B}, \mathrm{C}$ respectively. You answers must be accurate to 5 decimal places.
(iii) Again, using the sample means, standard deviations, and correlation matrix that you calculated in part (i), fill in the entries in the table below. The reported standard deviation and portfolio weights in a given row should be that of the frontier portfolio corresponding to the given mean. You can use Solver in Excel or any other method as use of formulae that you have derived in your Assignment\#2 or V.B. Scripts to solve for these portfolio weights.

| Mean | standard deviation | weight A | weight B | weight C |
| :--- | :--- | :--- | :--- | :--- |
| $2.2 \%$ |  |  |  |  |
| $2.0 \%$ |  |  |  |  |
| $1.8 \%$ |  |  |  |  |
| $1.6 \%$ |  |  |  |  |
| $1.4 \%$ |  |  |  |  |
| $1.2 \%$ |  |  |  |  |
| $1.0 \%$ |  |  |  |  |

(iv) Plot the portfolio frontier using the entries in the preceding table. Plot each of the individual companies inside the feasible set. Comment on the weights of the portfolios along the portfolio frontier, making sure that you include some discussion of the correlation among the 3 companies.
(v) Now let's run a horse race. You have decided to invest all the money in the portfolio on the frontier that gives average monthly return of $2.0 \%$. Suppose you make the investment based on these weights on January 1, 2014. How much is the return from January to March 2015 for this frontier portfolio? How much will be the return had you invested in an equal-weighted portfolio of the 3 stocks?
(vi) Suppose there is a riskless asset that has a monthly return of $0.5 \%$. Plot the resulting portfolio frontier and identify the tangent portfolio.

## Question \# 3: [Marks\#10 (5+5)]

(a) You have a 9-month American put option that is dollar dominated on British pounds. It is given that:
(i) The current exchange rate is 1.43 US dollars/pound
(ii) The strike price of the put is 1.56 US dollars/pound.
(iii) The volatility of the exchange rate is $\sigma=0.3$
(iv) The US dollar continuously compounded risk-free interest rate is $8 \%$.
(v) The British pound continuously compounded risk-free interest rate is $9 \%$.

Use a periodic binomial model and calculate the price of the put. Show all your calculations clearly with diagrams.
(b) A stock paying dividends and having an European options on this stock, you are provided the following data:
(i) The current stock price - Rs. 49.70.
(ii) The strike price of options - Rs. 50.00 .
(iii) The time to expiration - 6 months.
(iv) The risk-free rate - $3 \%$ annually.
(v) The continuous dividend yield rate - $2 \%$ annually.
(vi) The call price - $\$ 2.00$.
(vii) The put price - $\$ 2.35$.

By showing adequate calculations and reasoning, calculate the present value arbitrage profit per share that could be generated, given these conditions:

- Less than $\$ 0.20$
- At least $\$ 0.20$, but less than $\$ 0.40$
- At least $\$ 0.40$, but less than $\$ 0.60$
- At least $\$ 0.60$, but less than $\$ 0.80$
- At least $\$ 0.80$


## Question \# 4: [Marks\#10 (5+5)]

(a) Consider that two companies X and Y both wish to borrow Rs. $3^{11 / 2}$ crores for $71 / 2$ years and have been offered the interest rates as shown in table below. Remember MIBOR is Mumbai Inter-Bank Offered Rate, which fluctuates daily depending on demand and supply of money.

| Company | Fixed interest rate | Floating interest rate |
| :--- | :--- | :--- |
| X | $8.25 \%$ | 6-month MIBOR $+0.2 \%$ |
| Y | $9.05 \%$ | 6-month MIBOR $+0.8 \%$ |

Consider three different situations/cases:
(1) Company X is borrowing on its own from the market at the floating rate, which is MIBOR $+0.2 \%$, while company Y is borrowing independently at the fixed interest rate from the market, which is $9.05 \%$.

(2) Company A and B goes into an agreement as shown below:

(3) Company A and B goes into an agreement through an intermediary which is the financial institution, as shown below:


Answer the following questions:
(i) In comparison to case (1), what is the saving (in Rs.) for company X, considering case (2)?
(ii) In comparison to case (1), what is the saving (in Rs.) for company Y, considering case (2)?
(iii) In comparison to case (1), what is the total savings (in Rs.), for both company X and company Y, taken together, in case (2)?
(iv) In comparison to case (1), what is the saving (in Rs.), for company X considering case (3)?
(v) In comparison to case (1), what is the savings (in Rs.), for company Y considering case (3)?
(vi) What is the profit (in Rs.), for the financial institution in case (3)?
(b) Consider companies A and B have the following interest rate (\%) applicable to them respectively if they borrow in Japanese Yen (JY) and French Franc (FF).

|  | JY | FF |
| :--- | :--- | :--- |
| Company A | $6.0 \%$ | $11.5 \%$ |
| Company B | $8.2 \%$ | $13.3 \%$ |

Also assume company A has a comparative advantage if it wants to borrow in JY vis-à-vis FF, while it is just the opposite for company B , but in reality company A is required to borrow in FF , while company B is required to borrow in JY. Design a swap (you are required to draw and depict the flow of interest rates) such that both company A and B face currency risks while no currency risk is faced by the financial institution ( FI ), which acts as the intermediary.

