Answer#1

1.

a. 3-Period Moving Average: $F_{June} = (A_{March} + A_{April} + A_{May})/3 = (38+39+43)/3 = 40$
5-Period Moving Average: $F_{June} = (A_{January} + A_{February} + A_{March} + A_{April} + A_{May})/5$
$= (32+41+38+39+43)/5 = 38.6$

b. Naïve: $F_{June} = A_{May} = 43$

c. 3-Period Moving Average: $F_{July} = (A_{April} + A_{May} + A_{June})/3 = (39+43+41)/3 = 41$
5-Period Moving Average: $F_{July} = (A_{February} + A_{March} + A_{April} + A_{May} + A_{June})/5$
$= (41+38+39+43+41)/5 = 40.4$

Naïve: $F_{July} = A_{June} = 41$

d.

Month	Actual	3-Period Moving Average	Absolute Error	5-Period Moving Average	Absolute Error	Naïve	Absolute Error
January	32						
February	41					32	9
March	38					41	3
April	39	37	2			38	1
May	43	39.33	3.67			39	4
June	41	40	1	38.6	2.4	43	2

MAD(3-period moving average) = $\frac{\sum |Actual - Forecast|}{n}$ = (2+3.67+1)/3 = 2.22

MAD(5-period moving average)= $\frac{\sum |Actual - Forecast|}{n}$ = 2.4/1 = 2.4

MAD(Naïve) = $\frac{\sum |Actual - Forecast|}{n}$ = (9+3+1+4+2)/5 = 3.8

The 3-period moving average provides the best historical fit using the MAD criterion and would be better to use.

e.

Month	Actual	3-Period Moving Average	Squared Error	5-Period Moving Average	Squared Error	Naïve	Squared Error
January	32						
February	41					32	81
March	38					41	9
April	39	37	4			38	1
May	43	39.33	13.47			39	16
June	41	40	1	38.6	5.76	43	4

MSE(3-period moving average) = $\frac{\sum (Actual - Forecast)^2}{n-1}$ = (4+13.47+1)/2 = 9.24

MSE(5-period moving average)= $\frac{\sum (Actual - Forecast)^2}{n-1}$: Not possible to compute since there are not enough observations (i.e., n = 1).

MSE(Naïve) = $\frac{\sum (Actual - Forecast)^2}{n-1}$ = (81+9+1+16+4)/4 = 111/4 = 27.75

The 3-period moving average provides the best historical fit using the MSE criterion.

2.

Forecasts using $\alpha = 0.1$:

Week	Demand	Exponential Smoothing	Absolute Error
1	330	330	
2	350	330	20
3	320	332	12
4	370	330.8	39.2
5	368	334.72	33.28
6	343	338.048	4.952
		MAD:	21.89

Forecasts using $\alpha = 0.7$:

Week	Demand	Exponential Smoothing	Absolute Error
1	330	330	
2	350	330	20
3	320	344	24
4	370	327.2	42.8
5	368	357.16	10.84
6	343	364.748	21.748
		MAD:	23.88

Using $\alpha = 0.1$ provides a better historical fit based on the MAD criterion.

3.

Given: $T_4 = 20$, $A_5 = 90$, $S_4 = 85$

<u>Step 1:</u>
Smoothing the level of the series:
$S5 = \alpha A_5 + (1 - \alpha)(S_4 + T_4) = 0.20(90) + 0.80(85 + 20) = 102$

<u>Step 2:</u>
Smoothing the trend:
$T5 = \beta(S_5 - S_4) + (1 - \beta)T_4 = 0.10(102 - 85) + 0.90(20) = 19.7$

<u>Step 3:</u>
Forecast Including Trend
$FIT_6 = S_5 + T_5 = 102 + 19.7 = 121.7$

Answer#2

a)

The prices are not arbitrage-free. To show that Mary's portfolio yields arbitrage profit,

	Time 0	Time T			
		$S_T < 40$	$40 \le S_T < 50$	$50 \le S_T < 55$	$S_T \ge 55$
Buy 1 call Strike 40	− 11	0	$S_T - 40$	$S_T - 40$	$S_T - 40$
Sell 3 calls Strike 50	+ 18	0	0	$-3(S_T - 50)$	$-3(S_T - 50)$
Lend \$1	− 1	e^{rT}	e^{rT}	e^{rT}	e^{rT}
Buy 2 calls strike 55	− 6	0	0	0	$2(S_T - 55)$
Total	0	$e^{rT} > 0$	$e^{rT} + S_T - 40 > 0$	$e^{rT} + 2(55 - S_T) > 0$	$e^{rT} > 0$

Peter's portfolio makes arbitrage profit, because:

	Time-0 cash flow	Time-T cash flow
Buy 2 calls & sells 2 puts Strike 55	$2(-3 + 11) = 16$	$2(S_T - 55)$
Buy 1 call & sell 1 put Strike 40	$-11 + 3 = -8$	$S_T - 40$
Lend \$2	-2	$2e^{rT}$
Sell 3 calls & buy 3 puts Strike 50	$3(6 - 8) = -6$	$3(50 - S_T)$
Total	0	$2e^{rT}$

b)

The payoff at the contract maturity date is

$$\pi \times (1 - y\%) \times \text{Max}[S(T)/S(0), (1 + g\%)^T]$$
$$= \pi \times (1 - y\%) \times \text{Max}[S(1)/S(0), (1 + g\%)^1] \quad \text{because } T = 1$$
$$= [\pi/S(0)](1 - y\%)\text{Max}[S(1), S(0)(1 + g\%)]$$
$$= (\pi/100)(1 - y\%)\text{Max}[S(1), 103] \quad \text{because } g = 3 \ \& \ S(0) = 100$$
$$= (\pi/100)(1 - y\%)\{S(1) + \text{Max}[0, 103 - S(1)]\}.$$

Now, $\text{Max}[0, 103 - S(1)]$ is the payoff of a one-year European put option, with strike price \$103, on the stock index; the time-0 price of this option is given to be is \$15.21. Dividends are incorporated in the stock index (i.e., $\delta = 0$); therefore, $S(0)$ is the time-0 price for a time-1 payoff of amount $S(1)$. Because of the no-arbitrage principle, the time-0 price of the contract must be

$$(\pi/100)(1 - y\%)\{S(0) + 15.21\}$$
$$= (\pi/100)(1 - y\%) \times 115.21.$$

Therefore, the "break-even" equation is

$$\pi = (\pi/100)(1 - y\%) \times 115.21,$$

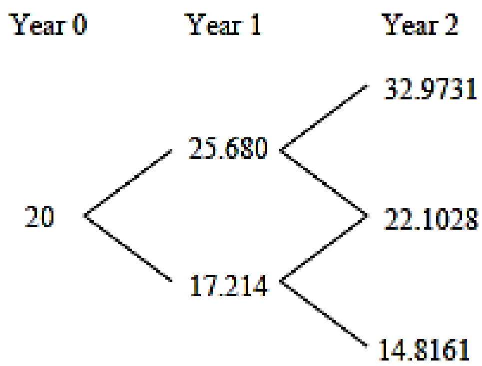
or

$$y\% = 100 \times (1 - 1/1.1521)\% = 13.202\%$$

Answer #3:

a)

First, we construct the two-period binomial tree for the stock price.



The calculations for the stock prices at various nodes are as follows:

$S_u = 20 \times 1.2840 = 25.680$
$S_d = 20 \times 0.8607 = 17.214$
$S_{uu} = 25.68 \times 1.2840 = 32.9731$
$S_{ud} = S_{du} = 17.214 \times 1.2840 = 22.1028$
$S_{dd} = 17.214 \times 0.8607 = 14.8161$

The risk-neutral probability for the stock price to go up is

$$p^* = \frac{e^{rh} - d}{u - d} = \frac{e^{0.05} - 0.8607}{1.2840 - 0.8607} = 0.4502.$$

Thus, the risk-neutral probability for the stock price to go down is 0.5498.

If the option is exercised at time 2, the value of the call would be
$C_{uu} = (32.9731 - 22)_+ = 10.9731$
$C_{ud} = (22.1028 - 22)_+ = 0.1028$
$C_{dd} = (14.8161 - 22)_+ = 0$

If the option is European, then $C_u = e^{-0.05}[0.4502C_{uu} + 0.5498C_{ud}] = 4.7530$ and $C_d = e^{-0.05}[0.4502C_{ud} + 0.5498C_{dd}] = 0.0440$.
But since the option is American, we should compare C_u and C_d with the value of the option if it is exercised at time 1, which is 3.68 and 0, respectively. Since $3.68 < 4.7530$ and $0 < 0.0440$, it is not optimal to exercise the option at time 1 whether the stock is in the up or down state. Thus the value of the option at time 1 is either 4.7530 or 0.0440.

Finally, the value of the call is
$C = e^{-0.05}[0.4502(4.7530) + 0.5498(0.0440)] = 2.0585.$

b)

$$C(S,K,\sigma,r,T,\delta) = Se^{-\delta T}N(d_1) - Ke^{-rT}N(d_2) \qquad (12.1)$$

with

$$d_1 = \frac{\ln(S/K) + (r - \delta + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \qquad (12.2a)$$

$$d_2 = d_1 - \sigma\sqrt{T} \qquad (12.2b)$$

Because $S = 20$, $K = 25$, $\sigma = 0.24$, $r = 0.05$, $T = 3/12 = 0.25$, and $\delta = 0.03$, we have

$$d_1 = \frac{\ln(20/25) + (0.05 - 0.03 + \frac{1}{2}0.24^2)0.25}{0.24\sqrt{0.25}} = -1.75786$$

and

$$d_2 = -1.75786 - 0.24\sqrt{0.25} = -1.87786$$

Using the Cumulative Normal Distribution Calculator, we obtain $N(-1.75786) = 0.03939$ and $N(-1.87786) = 0.03020$.

Hence, formula (12.1) becomes

$$C = 20e^{-(0.03)(0.25)}(0.03939) - 25e^{-(0.05)(0.25)}(0.03020) = 0.036292362$$

Cost of the block of 100 options = $100 \times 0.0363 = \$3.63$.

<u>OBJECTIVE</u>

1		5	A+B
2		6	E
3		7	C
4		8	D

9	Gold
10	The average return on Gold is much less than on the stock market
11	14% and 167.25
12	The optimal amount to invest in gold would drop

13	B
14	A
15	B