## NPTEL <br> DEPARTMENT OF INDUSTRIAL AND MANAGEMENT ENGINEERING <br> IIT KANPUR <br> QUANTITATIVE FINANCE <br> END-TERM EXAMINATION (2015 JULY-AUG ONLINE COURSE)

## READ THE INSTRUCTIONS VERY CAREFULLY

1) Time duration is 2 hours
2) The total marks is 150 .
3) This question paper consists of TWO sections.
4) Section\#1-Subjective: 2 questions having 2 subparts each. There is step marking for all questions. You have to show all working and calculations to get the full marks. If your final answer is incorrect even though your approach is right, marks will be provided for working shown. There is no negative marking in this section.
5) Section\#2-Objective: 30 questions. Correct answer:4 marks. Incorrect answer: -1 marks. No marks for questions left unattended.
6) Write all answers in the answer sheets provided. Do not use separate answer sheets for objective questions. Please write the option number (A, B, C.... etc.) very clearly in the same answer sheets provided and give explanation wherever it is asked.
7) Questions are self-explanatory. Make necessary assumptions wherever required.
8) Only non-Programmable calculators are allowed.

## PART\#1 - Subjective

Question \# 1: [Marks\#15 (10+5)]
(a) You have the following data for prices (per stock) and amount of investments for two stocks given for 10 consecutive days. All the data is in Rs.
$\mathrm{S}_{1}=55.25,55.50,56.00,55.00,55.25,57.25,57.50,58.00,57.00,56.50$
$\alpha_{1}=20.00,20.00,20.00,20.00,20.00,20.00,20.00,20.00,20.00,20.00$
$\mathrm{S}_{2}=100.50,102.00,101.75,101.50,100.00,102.50,102.50,102.25,102.00,101.75$
$\alpha_{2}=80.00,80.00,80.00,80.00,80.00,80.00,80.00,80.00,80.00,80.00$
Show how you would find the $99 \%$ VaR for the portfolio (formed with one option of stock \# 1 and one option of stock \# 2) for 10 days.
(b) The NPTEL Home Tutor Solutions Pvt. Ltd. help customers to find private tuitions and coaching centers in Kanpur city as well as online tutors. This supply business is competitive, and the ability to deliver talented as well as well-educated tutors promptly is a big factor in getting new customers and maintaining old ones. The manager of the company wants to be certain that enough tutors are available at hand to meet demand promptly. Therefore, the manager wants to be able to forecast the demand for requirement of tutors during the next month. From the records of previous orders, management has accumulated the following data for the past 10 months:

| Month | Jan. | Feb. | Mar. | Apr. | May | Jun. | Jul. | Aug. | Sep. | Oct. |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Orders | 120 | 90 | 100 | 75 | 110 | 50 | 75 | 130 | 110 | 90 |

(i) Compute the monthly demand forecast for February through November using the naive method.
(ii) Compute the monthly demand forecast for April through November using a 3month moving average.
(iii) Compute the monthly demand forecast for June through November using a 5month moving average.
(iv) Compute the monthly demand forecast for April through November using a 3month weighted moving average. Use weights of $0.5,0.33$, and 0.17 , with the heavier weights on the more recent months.
(v) Compute the mean absolute deviation for June through October for each of the methods used. Which method would you use to forecast demand for November?

## Question \# 2: [Marks\#15 (10+5)]

(a) Consider a forward start option which, 1 year from today, will give its owner a 1-year European call option with a strike price equal to the stock price at that time.
You are given:
(i) The European call option is on a stock that pays no dividends.
(ii) The stock's volatility is $30 \%$.
(iii) The forward price for delivery of 1 share of the stock 1 year from today is 100 .
(iv) The continuously compounded risk-free interest rate is $8 \%$.

Under the Black-Scholes framework, determine the price today of the forward start option. Show your working clearly. Make necessary assumptions for data. You may keep the answer in form of an expression. Marks will be allocated only for correct formulation of the problem.
(Given, the values from the Z table are as follows: $\mathrm{N}(0.417)=0.661, \mathrm{~N}(0.117)=0.546$. The variables used are in their standard form)
(b) You are considering the purchase of a 3-month 41.5-strike American call option on a nondividend paying stock. You are given:
(i) The Black-Scholes framework holds.
(ii) The stock is currently selling for 40.
(iii) The stock's volatility is $30 \%$.
(iv) The current call option delta is 0.5 .
(v) Use your common sense to get the required values of $\mathrm{N}\left(d_{1}\right)$ and $\mathrm{N}\left(d_{2}\right)$ that is required from the Z table.
Determine the current price of the option. Show your calculations. Step marks will be awarded for correct formulation of problem.
(A) $20-20.453 \int_{-\infty}^{0.15} e^{-x^{2} / 2} \mathrm{~d} x$
(B) $20-16.138 \int_{-\infty}^{0.15} e^{-x^{2} / 2} \mathrm{~d} x$
(C) $20-40.453 \int_{-\infty}^{0.15} e^{-x^{2} / 2} \mathrm{~d} x$
(D) $16.138 \int_{-\infty}^{0.15} e^{-x^{2} / 2} \mathrm{~d} x-20.453$
(E) $40.453 \int_{-\infty}^{0.15} e^{-x^{2} / 2} \mathrm{~d} x-20.453$

## PART\#II - Objective

1) Markowitz considered the objective function of the form
(A) $\min \frac{1}{2} \sum_{i=1}^{n} w_{i} w_{j} \sigma_{i, j}$
(B) $\max \frac{1}{2} \sum_{i=1}^{n} w_{i} w_{j} \sigma_{i, j}$
(C) $\min \sum_{i=1}^{n} w_{i} \bar{r}_{i}$
(D) $\gamma^{*} \max \sum_{i=1}^{n} w_{i} \bar{r}_{i}+(1-\gamma) * \min \frac{1}{2} \sum_{i=1}^{n} w_{i} w_{j} \sigma_{i, j}$
2) Explain very briefly. When utility functions are quadratic, the returns are
(A) normal
(B) exponential
(C) gamma
(D) poisson
3) You have three (3) financial assets with the following set of information

| Asset (i) | A | B | C | Average return |
| :--- | :--- | :--- | :--- | :--- |
| A | 4.00 | 3.00 | 4.00 | 4.00 |
| B | 3.00 | 9.00 | 6.00 | 6.00 |
| C | 4.00 | 6.00 | 16.00 | 8.00 |

If SS is allowed, then at MVP the weights of $A, B$ and $C$ are
(A) $(6 / 12,4 / 12,3 / 12)$
(B) $(3 / 12,6 / 12,4 / 12)$
(C) $(4 / 12,3 / 12,6 / 12)$
(D) none of the above
4) For, problem \# 3, given above, the return at MVP is
(A) 4.00
(B) 6.67
(C) 6.83
(D) none of the above
5) For, problem \# 3, given above, the variance, V(MVP), at MVP is
(A) $\mathrm{V}(\mathrm{MVP})<4.00$
(B) $4.00 \leq \mathrm{V}(\mathrm{MVP})<9.00$
(C) $9.00 \leq \mathrm{V}(\mathrm{MVP})<16.00$
(D) $\mathrm{V}(\mathrm{MVP}) \geq 16.00$
6) If we have two (2) assets, A and B , with the following expected returns and variance as $\left(\bar{r}_{A}=10.00, \sigma_{A}^{2}=9.00\right)$ and $\left(\bar{r}_{B}=15.00, \sigma_{B}^{2}=25.00\right)$ respectively, and with $\rho_{\mathrm{AB}}=-1.00$, then we can ensure that for a certain combination of $A$ and $B$ we have zero variance and some returns. What is that returns
(A) 20.00
(B) 15.00
(C) 10.00
(D) none of the above
7) Explain very briefly. For $(\mathrm{n}+1)$ financial assets, with $n$ of them being risky and the $(\mathrm{n}+1)^{\text {th }}$ being the risk free interest one, then AB line signifies: (Next Page)

(A) Feasible set
(B) efficient frontier
(C) Minimum variance loci
(D) none of the above
8) Point $\mathrm{P}\left(\mathrm{l}(\mathrm{QP})=l\left(0.5^{*} \mathrm{AQ}\right)\right)$ signifies a combination of

(A) $-150 \%$ in Q and +50 in A
(B) $-150 \%$ in A and +50 in Q
(C) $-50 \%$ in A and +150 in Q
(D) $-50 \%$ in Q and +150 in A
9) The primary risk involved in an "uncovered interest arbitrage" transaction is:
(A) The foreign currency will fluctuate
(B) The foreign interest rate will fluctuate
(C) The US interest rate will fluctuate
(D) All of the above
10) Which of the following is not possible when two securities are positively correlated:
(A) Asset A's mean return is negative while asset B's is positive
(B) Asset A's return is sometimes below its mean when asset B's is above its mean
(C) Asset A's mean return is negative while asset B's mean return is also negative
(D) All are possible
11) If you are given the following data, where the $t, i$ and $m$ signify the time, asset and the market respectively, then $\beta$ for the asset is

| t | Average return(i,t) | Average return(m,t) |
| :--- | :--- | :---: |
| 1 | 10 | 12 |
| 2 | 15 | 12 |
| 3 | 12 | 10 |
| 4 | 11 | 15 |
| 5 | 09 | 14 |

(A) $\beta<0.25$
(B) $0.25 \leq \beta<0.75$
(C) $0.75 \leq \beta<0.85$
(D) none of the above
12) Beta is also expressed as
(A) $\beta=\rho_{i, m}\left(\frac{\sigma_{i, m}}{\sigma_{m}^{2}}\right)$
(B) $\beta=\rho_{i, m}\left(\frac{\sigma_{i}^{2}}{\sigma_{m}^{2}}\right)$
(C) $\beta=\rho_{i, m}\left(\frac{\sigma_{i}}{\sigma_{m}}\right)$
(D) $\beta=\rho_{i, m}\left(\frac{\sigma_{m}}{\sigma_{i}}\right)$
13) For the single index model the following formulae is correct
(A) $\sigma_{P}^{2}=\beta_{P}^{2} \sigma_{m}^{2}+\sum_{i=1}^{n} w_{i} \sigma_{\varepsilon(i)}$
(B) $\sigma_{P}^{2}=\beta_{P}^{2} \sigma_{P}^{2}+\sum_{i=1}^{n} w_{i}^{2} \sigma_{\varepsilon(i)}^{2}$
(C) $\sigma_{P}^{2}=\beta_{P}^{2} \sigma_{m}^{2}+\sum_{i=1}^{n} w_{i}^{2} \sigma_{\varepsilon(i)}^{2}$
(D) $\sigma_{P}^{2}=\beta_{P}^{2} \sigma_{m}^{2}+\sum_{i=1}^{n} w_{i} \sigma_{\varepsilon(i)}^{2}$
14) If we invest in equal proportion in $n$ stocks and the single index model is true, then
(A) $\sigma_{P}^{2}=\beta_{P}^{2} \sigma_{P}^{2}+\sum_{i=1}^{n} \frac{1}{n} \sigma_{\varepsilon(i)}$
(B) $\sigma_{P}^{2}=\beta_{m}^{2} \sigma_{P}^{2}+\frac{1}{n^{2}} \sum_{i=1}^{n} \sigma_{\varepsilon(i)}^{2}$
(C) $\sigma_{P}^{2}=\beta_{P}^{2} \sigma_{p}^{2}+\frac{1}{n^{2}} \sum_{i=1}^{n} \sigma_{\varepsilon(i)}^{2}$
(D) none of the above
15) Given the data below, in which combination of assets (taken ONLY two assets at a time in equal proportions, with your main aim being to minimize the risk (standard deviation or variance) of the portfolio thus formed with these two (2) assets you consider) will you invest, considering that the single index model is true. Consider you are provided with the following values, $R_{f}=5$, $\bar{R}_{m}=12.00, \sigma_{m}^{2}=10$ and $\rho_{\mathrm{i}, \mathrm{j}}(\mathrm{i} \neq \mathrm{j}, \mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E})=+0.5$. For ease of calculations we are omitting the units of return and variance.

| Security (i) | $\bar{R}_{i}$ | $\beta_{i}$ | $\sigma_{i}$ | $\sigma_{\varepsilon_{i}}^{2}$ |
| :--- | :--- | :--- | :--- | :--- |
| A | 15 | 1.0 | 10 | 30 |
| B | 12 | 1.5 | 10 | 20 |
| C | 11 | 2.0 | 11 | 40 |
| D | 13 | 0.8 | 5 | 10 |
| E | 10 | 1.0 | 7 | 20 |

16) For, problem \# 15, given above, in which combination of assets (taken ONLY two assets at a time in equal proportions, with your main aim being to maximize the return of the portfolio thus formed with these two (2) assets you consider) will you invest,
(A) B \& C
(B) A \& D
(C) B \& D
(D) C \& E
17) For, problem \# 15, given above, in which combination of assets (taken ONLY two assets at a time in equal proportions, with your main aim being to minimize the ratio of return to risk (risk now being beta) of the portfolio thus formed with these two (2) assets you consider) will you invest,
(A) A \& E
(B) B \& C
(C) C \& D
(D) B \& D
18) For a long forward with a delivery price of 65 (INR), if the price of the asset at contract maturity is 75 (INR), then the payoff in INR is?
(A) +20
(B) +10
(C) 0
(D) -10
(E) -20
19) For a short forward with a delivery price of 100 (INR), if the price of the asset at contract maturity is 110 (INR), then the payoff in INR is?
(A) +20
(B) +10
(C) 0
(D) -10
(E) -20
20) For a long forward with a delivery price of 105 (INR), if the price of the asset at contract maturity is 85 (INR), then the payoff in INR is?
(A) +20
(B) +10
(C) 0
(D) -10
(E) - 20
21) If you sell one call option (option price $=5$ strike price $=75$ ) and buy a put option (option price $=$ 10 , strike price $=125$ ). Consider the time to maturity for both the options is same, which is nine months. Then when the spot price of the asset at time to maturity is 100 , then the combined/total payoff from the two options, given that all prices are in INR, in INR, is
(A) -10
(B) 0
(C) +5
(D) +10
(E) None of the above
22) If the current spot price for a financial asset is 1020 (INR) and the risk free interest rate is $4.5 \%$ per annum (continuously compounded and considered to be fixed throughout the time till maturity of the derivative), then the forward price, in INR, for the corresponding derivative (based on this financial asset), which has a time to maturity of one and a half year is approximately
(A) 1090
(B) 1090.5
(C) 1091
(D) 1091.5
(E) 1092
23) Suppose you buy a ten year $\$ 1000$ face value zero coupon bond whose yield to maturity (annual compounding) is 7 percent. You sell the bond exactly two years later, when the yield to maturity is 10 percent. What is the price change per $\$ 1000$ bond?
(A) $+\$ 73.67$
(B) $+\$ 31.84$
(C) $-\$ 41.84$
(D) $-\$ 73.67$
24) (This question has Multiple Answers) Identify the first two steps in a profitable arbitrage, given the following:
Yield on U.K. government one-year note: $8 \%$
Yield on U.S. government one-year note: 5\%
Exchange rate (spot): 1.60 USD/Pound
Exchange rate (one year forward): 1.70 USD/Pound
(A) Sell short US securities
(B) Sell USD in spot foreign exchange market for pounds
(C) Sell short UK securities
(D) Sell pounds in spot foreign exchange market for USD
(E) There is no arbitrage
25) A regression model is used to forecast sales based on advertising dollars spent. The regression line is $\mathrm{y}=500+35 \mathrm{x}$ and the coefficient of determination is .90 . Which is the best statement about this forecasting model?
A) The correlation between sales and advertising is positive.
B) The coefficient of correlation between sales and advertising is 0.81 .
C) For every $\$ 35$ spent on advertising, sales increase by $\$ 1$
D) Even if no money is spent on advertising, the company realizes $\$ 35$ of sales
26) Given last period's forecast of 65 , and last period's demand of 62 , what is the simple exponential smoothing forecast with an alpha of 0.4 for the next period?
A) 65
B) 63.2
C) 63.8
D) 62
27) Alan has just entered into a derivative position with a dealer. The dealer makes a positive payoff when the price of underlying asset is less than $\$ 35$ and higher than $\$ 45$ at expiration. Which of the followings describes the option strategy that the dealer has entered into?
I. 35-45 purchased strangle
II. $35-45$ written strangle
III. 35-40-45 butterfly-spread
IV. 35-45 Bull call spread
(A) I, III
(B) II, III
(C) II, IV
(D) III, IV
(E) None of these
28) The current (spot) rate for corn is 1.60 per bushel. The 6 month forward price is $\$ 1.50$ per bushel. The continuously compounded annual rate is $\mathrm{r}=.035$. Farmer Brown, has total fixed and variable costs of 1.44 per bushel, and plans to produce 100,000 bushels for $\$ 144,000$. A six month ( $\mathrm{T}=.5$ ) put with a strike price of 1.52 per bushel is available at a price of 0.12 . What are the minimum and maximum profits for Farmer Brown in six months if he is hedged with a purchase of this put? A) Minimum $=-4,212$, Maximum $=19,678$
B) Minimum $=6222$, Maximum $=19,678 \mathrm{C}$ )

Minimum $=-4$, 212, no maximum
D) Minimum $=-6,242$, no Maximum E) none of the above
29) The $S \& R$ index currently has a price of 1300 . The price of a three month 1320 -strike put is 81.41 . The annual interest rate is $4 \%$ compounded continuously. A buys this put, and $B$ enters into a long forward contract. In three months A and B have the same profit. What is the price of the index in three months?
A) 1310
B) 1297
C) 1289
D) 1291
E) 1275
30) The president of State University wants to forecast student enrollments for this academic year based on the following historical data: 5 years ago; 15,000, 4 years ago; 16,000, 3 years ago; $18,000,2$ years ago; 20,000, Last year; 21,000. What is the forecast for this year using exponential smoothing with $\alpha=0.4$, if the forecast for two years ago was 16,000 ?
A) 17600
B)17850
C) 19420
D)18960
E) 19240

