## INDUSTRIAL AND MANAGEMENT ENGINEERING DEPARTMENT, IIT KANPUR QUANTITATIVE FINANCE <br> ASSIGNMENT-5 (2015 JULY-AUG ONLINE COURSE)

## NOTE THE FOLLOWING

1) There are five questions and you are required to answer all of them.
2) Deadline for submission is Saturday; $15^{\text {th }}$ August, 2015
3) The total marks is 50 .
4) To get full credit do your calculations carefully.

Question 1:
a) At time t, 3 M borrows $¥ 12.8$ billion at an interest rate of 1.2 percent, paid semiannually, for a period of two years. It then enters into a two-year yen/dollar swap with Bankers Trust (BT) on a notional principal amount of $\$ 100$ million ( $¥ 12.8$ billion at the current spot rate). Every six months, 3M pays BT U.S. dollar LIBOR6, while BT makes payments to 3 M of 1.3 percent annually in yen. At maturity, BT and 3 M reverse the notional principals
(i) Assume that LIBOR6 (annualized) and the $¥ / \$$ exchange rate evolve as follows. Calculate the net dollar amount that 3M pays to BT ("-") or receives from BT ("+") each six-month period.

| Time (months) | LIBOR6 | ¥/\$ (spot) | Net \$ receipt (+)/payment (-) |
| :--- | :--- | :--- | :--- |
| t | $5.7 \%$ | 128 |  |
| $\mathrm{t}+6$ | $5.4 \%$ | 132 |  |
| $\mathrm{t}+12$ | $5.3 \%$ | 137 |  |
| $\mathrm{t}+18$ | $5.9 \%$ | 131 |  |
| $\mathrm{t}+24$ | $5.8 \%$ | 123 |  |

(ii) What is the all-in dollar cost of 3M's loan?
(iii) Suppose 3 M decides at $\mathrm{t}+18$ to use a six-month forward contract to hedge the $t+24$ receipt of yen from BT. Six-month interest rates (annualized) at $t+18$ are $5.9 \%$ in dollars and $2.1 \%$ in yen. With this hedge in place, what fixed dollar amount would 3 M have paid (received) at time $\mathrm{t}+24$ ? How does this amount compare to the $\mathrm{t}+24$ net payment computed in part a?
(iv) Does it make sense for 3 M to hedge its receipt of yen from BT? Explain.
b) Suppose that IBM would like to borrow fixed-rate yen, whereas Korea Development Bank (KDB) would like to borrow floating-rate dollars. IBM can borrow fixed-rate yen at 4.5 percent or floating-rate dollars at LIBOR +0.25 percent. KDB can borrow fixedrate yen at 4.9 percent or floating-rate dollars at LIBOR +0.8 percent.
(i) What is the range of possible cost savings that IBM can realize through an interest rate/currency swap with KDB? The cost to each party of accessing either the fixed-rate yen or the floating-rate dollar market for a new debt issue is as follows:

## Borrower

Korea Development Bank IBM

Fixed-Rate Yen Available
4.9\%
4.5\%
$0.4 \%$

Floating-Rate Dollars Available
LIBOR + 0.80\%
LIBOR $+0.25 \%$
$0.55 \%$
(ii) Assuming a notional principal equivalent to $\$ 125$ million, and a current exchange rate of $¥ 105 / \$$, what do these possible cost savings translate into in yen terms?
(iii) Redo Parts a and b assuming that the parties use Bank of America, which charges a fee of 8 basis points to arrange the swap.

## Question 2:

a) (i) Consider you undertake the following transactions:
(AI): Buy one call option for Rs. 5, the strike price of which is Rs. 50 .
(AII): Sell two call options for Rs. 10 each, the strike price of each of the option is Rs. 75.
(AIII): Buy one call option for Rs. 15, the strike price of which is Rs. 100.
Then:
What is the payoff matrix (depending on the stock price, $\mathrm{S}_{\mathrm{T}}$ ) for the above three transactions you undertook? Draw the cumulative payoff graph vs $\mathrm{S}_{\mathrm{T}}$ for the above three transactions you undertook.
(ii) Consider you undertake the following transactions:
(AI): Sell a call option for Rs. 10, the strike price of which is Rs. 100.
(AII): Sell a put option for Rs. 15, the strike price of which is Rs. 100.
(AIII): Buy a call option for Rs. 20, the strike price of which is Rs. 150 .
(AIV): Buy a put option for Rs. 25, the strike price of which is Rs. 150.
Then:
What is the payoff matrix (depending on the stock price, $\mathrm{S}_{\mathrm{T}}$ ) for the above four transactions you undertook?
b) Consider that two companies A and B both wish to borrow Rs. 1 crore for five years and have been offered the interest rates as shown in table below. Remember MIBOR is Mumbai InterBank Offered Rate, which fluctuates daily depending on demand and supply of money.

| Company | Fixed interest rate | Floating interest rate |
| :--- | :--- | :--- |
| A | $10.0 \%$ | 6-month MIBOR $+0.3 \%$ |
| B | $11.2 \%$ | 6-month MIBOR $+1.0 \%$ |

Consider three different situations/cases:
(1) Company A is borrowing on its own from the market at the floating rate, which is MIBOR $+0.3 \%$, while company $B$ is borrowing independently at the fixed interest rate from the market, which is $11.2 \%$.

(2) Company A and B goes into an agreement as shown below:

(3) Company A and B goes into an agreement through an intermediary which is the financial institution, as shown below:


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Answer the following questions:
a) In comparison to case (1), what is the saving for company A, if we consider case (2)?
b) In comparison to case (1), what is the saving for company B, if we consider case (2)?
c) In comparison to case (1), what is the total savings, for both company A and company B, taken together, in case (2)?
d) In comparison to case (1), what is the saving, for company A if we consider case (3)?
e) In comparison to case (1), what is the savings, for company B if we consider case (3)?
f) What is the profit, for the financial institution in case (3)?
g) What is the total savings, for company A, company B and the financial institution in case (3)?

Question 3:
a) Assume a standard 3-period CRR binomial model. The price of the stock is currently $\$ 100$. The risk-free interest rate with continuous compounding is $6 \%$ per annum. Over the next three 4 month periods, the stock is expected to go up by $8 \%$ or go down by $7 \%$ in each period.
(i) What is the value of a one-year European call with strike price $\$ 103$ ?
(ii) What is the value of a one-year European put with strike price $\$ 103$ ?
(iii) Verify the Put-Call parity for the European call and the European put.
b) Assume a 3-period Cox-Ross-Rubinstein model. The annual interest rate with continuous compounding is $r=0.06$. The volatility of the stock is $\sigma=0.2$ with a price of $\mathrm{S}(0)=100$. Furthermore, there exists an American Put with maturity date $\mathrm{T}=1$ und strike $\mathrm{K}=90$.
(i) Calculate the risk-neutral probability and the stock prices at each node in the binomial tree (correct up to 2 decimal places after the decimal point).
(ii) Calculate the value of the American Put for all nodes in the tree.
(iii) What is the optimal stopping time? Justify your answer.

Question 4:
a) Suppose that put options on a stock with strike prices 35 and 40 cost 5 and 10 respectively. How can the options be used to create (i) a bull spread, (ii) a bear spread? Construct a table that shows the profit/payoff from both spreads.
b) Consider you are given the following table which summarizes the effect of few variables on option

| Variable | European Call | European Put |
| :--- | :--- | :--- |
| Stock price | + (positive) | - (negative) |
| Strike price | - (negative) | + (positive) |
| Time to expiration | ? (unknown) | ? (unknown) |
| Volatility | + (positive) | + (positive) |
| Risk free interest rate | + (positive) | - (negative) |

A diagonal spread is constructed by buying a call with a strike price $\mathrm{K}_{2}$ and exercise date $\mathrm{T}_{2}$ and selling a call with strike price $K_{1}$ and exercise date $T_{1}$, where $T_{2}>T_{1}$. Draw two separate diagrams showing the payoff when (i) $\mathrm{K}_{2}>\mathrm{K}_{1}$ and (ii) $\mathrm{K}_{2}<\mathrm{K}_{1}$.

Question 5:
a) A strangle is formed by
(i) Short put and a short call
(ii) Long put and a short call
(iii) Long put and a long call
(iv) Short put and long call
(v) None of the above
b) For a long forward with a delivery price of 65 (INR), if the price of the asset at contract maturity is 75 (INR), then the payoff in INR is?
(i) +20
(ii) +10
(iii) 0
(iv) -10
(v) -20
c) For a short forward with a delivery price of 100 (INR), if the price of the asset at contract maturity is 110 (INR), then the payoff in INR is?
(i) +20
(ii) +10
(iii) 0
(iv) -10
(v) -20
d) For a long forward with a delivery price of 105 (INR), if the price of the asset at contract maturity is 85 (INR), then the payoff in INR is?
(i) +20
(ii) +10
(iii) 0
(iv) -10
(v) -20
e) If you sell one call option (option price $=5$ strike price $=75$ ) and buy a put option (option price $=10$, strike price $=125$ ). Consider the time to maturity for both the options is same, which is nine months. Then when the spot price of the asset at time to maturity is 100 , then the combined/total payoff from the two options, given that all prices are in INR, in INR, is
(i) -10
(ii) 0
(iii) +5
(iv) +10
(v) None of the Above
f) Assume that we have a three period CRR model with initial stock price $\mathrm{S}=\$ 150$, interest rate $\mathrm{r}=0: 05$ and volatility $3 / 4=0: 2$.
(i) What is the value of an American Put with strike $\$ 150$, which matures in 6 months?
(ii) What is the value of an American Call with strike \$150, which matures in 6 months?
(iii) Verify that the following inequalities hold

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S-K \leq C_{A}-P_{A} \leq S-K e^{-r T}
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