## NPTEL

## INDUSTRIAL AND MANAGEMENT ENGINEERING DEPARTMENT, IIT KANPUR

## QUANTITATIVE FINANCE

## ASSIGNMENT-4 (2015 JULY-AUG ONLINE COURSE)

1. a) According to the information furnished are you of the opinion the contract signed is normal? Is so why and if not then also why? Give valid explanations and show calculations for your answer.
Answer: (i) $I=5 e^{-0.08 \frac{4}{12}}+5 e^{-0.08 \frac{8}{12}}=9.61$
$F_{0}=\left(S_{0}-I\right) e^{r T}=(130-9.61) e^{0.08 \frac{11}{12}}=129.50$
Now $\quad F_{0}=\left(S_{0}-I\right) e^{r T}=K \quad$ for no arbitrageur, i.e. we should have $f=S_{0}-I-K e^{-r T}=0$. Clearly this is not the case here and there would be arbitrage.
(ii) If such a contract was available to you what would your strategy be and why? Explain.
Answer: $F_{0}=\left(S_{0}-I\right) e^{r T}<K$, as $\mathrm{F}_{0}=129.50$ and $\mathrm{K}=135$, hence arbitrage will take place. Strategy should be, go SHORT on contact.
(iii) What change in the value of delivery price will make this abnormality go and make this contract a normal one?
Answer: Rs. $(135.00-129.50=5.50)$, i.e., decrease of Rs. 5.50 in the delivery price K.
b) $\sigma_{S}^{2}=\frac{1}{8} \sum_{i=1}^{8}\left(S_{i}-\bar{S}\right)^{2}=0.115, \sigma_{F}^{2}=\frac{1}{8} \sum_{i=1}^{8}\left(F_{i}-\bar{F}\right)^{2}=0.180$,
$\rho=\frac{1}{8} \sum_{i=1}^{8} \frac{\left(S_{i}-\bar{S}\right)\left(F_{i}-\bar{F}\right)}{\sigma_{S} \sigma_{F}}=0.971, h=\rho \frac{\sigma_{S}}{\sigma_{F}}=0.777$
As one future contract is worth 25,000 barrels of fuel, hence the number of contracts the company should sign is $0.777 \frac{10000000}{25000}=310.8 \approx 311$
$\begin{array}{lllll}\text { 2. a) Range Payoff from Total Of Price } & \text { Long Call Option } & \text { Short } & \text { Call } \\ \text { OptionPayoff } & & & \\ 90\left(\mathrm{~S}_{\mathrm{T}}\right)<100\left(\mathrm{~K}_{1}\right)<120\left(\mathrm{~K}_{2}\right) & -10 & +5 & -5 & \\ 100\left(\mathrm{~K}_{1}\right)<105\left(\mathrm{~S}_{\mathrm{T}}\right)<120\left(\mathrm{~K}_{2}\right)(105-100)-10+5 & 0 & & \\ 100\left(\mathrm{~K}_{1}\right)<110\left(\mathrm{~S}_{\mathrm{T}}\right)<120\left(\mathrm{~K}_{2}\right)(110-100)-10+5 & +5 & & \end{array}$

(b)(ii) The investor makes a profit if the price of the stock on the expiration date is less than $\$ 37$. In these circumstances the gain from exercising the option is greater than $\$ 3$. The option will be exercised if the stock price is less than $\$ 40$ at the maturity of the option. The variation of the investor's profit with the stock price in Figure:

(iii) Figure below shows the variation of the trader's position with the asset price. We can divide the alternative asset prices into three ranges:
(a)

When the asset price less than $\$ 40$, the put option provides a payoff of $40-S_{T}$ and the call option provides no payoff. The options cost $\$ 7$ and so the total profit is $33-S_{T}$.
(b)

When the asset price is between $\$ 40$ and $\$ 45$, neither option provides a payoff. There is a net loss of \$7.
(c)

When the asset price greater than $\$ 45$, the call option provides a payoff of $S_{T}-45$ and the put option provides no payoff. Taking into account the $\$ 7$ cost of the options, the total profit is $S_{T}-52$.
The trader makes a profit (ignoring the time value of money) if the stock price is less than $\$ 33$ or greater than $\$ 52$. This type of trading strategy is known as a strangle.

3. (b) The investor makes a profit if the price of the stock is below $\$ 59$ on the expiration date. If the stock price is below $\$ 55$, the option will not be exercised, and the investor makes a profit of $\$ 4$. If the stock price is between $\$ 55$ and $\$ 59$, the option is exercised and the investor makes a profit between $\$ 0$ and $\$ 4$. The variation of the investor's profit with the stock price is as shownin Figure

4. A similar example has been taken from the book by J.C. HULL

Operation of margins for a long position in two gold futures contracts. The initial margin is $\$ 6,000$ per contract, or $\$ 12,000$ in total; the maintenance margin is $\mathrm{S} 4,500$ per contract, or $\$ 9.000$ in total. The contract is entered into on Day 1 at $\$ 1,250$ and closed out on Day 16 at $\$ 1226,90$.

| Day | Trade <br> price $(\$)$ | Settiement <br> price $(\$)$ | Daily <br> gain $(\$)$ | Cimulative <br> gain $(\$)$ | Margin account <br> balance $(\$)$ | Margin <br> cail $(\$)$ |
| ---: | :---: | ---: | ---: | :---: | :---: | :---: |
| 1 | $1,250.00$ |  |  |  | 12,000 |  |
| 1 |  | $1,241.00$ | $-1,800$ | $-1,800$ | 10,200 |  |
| 2 |  | $1,238.30$ | -540 | $-2,340$ | 9,660 |  |
| 3 |  | $1,244.60$ | 1,260 | $-1,080$ | 10,920 |  |
| 4 |  | $1,241.30$ | -660 | $-1,740$ | 10,260 |  |
| 5 |  | $1,240.10$ | -240 | $-1,980$ | 10,020 |  |
| 6 |  | $1,236.20$ | -780 | $-2,760$ | 9,240 |  |
| 7 |  | $1,229.90$ | $-1,260$ | $-4,020$ | 7,980 | 4,020 |
| 8 |  | $1,230.80$ | 180 | $-3,840$ | 12,180 |  |
| 9 |  | $1,225,40$ | $-1,080$ | $-4,920$ | 11,100 |  |
| 10 |  | $1,228.10$ | 540 | $-4,380$ | 11,640 |  |
| 11 |  | $1,211.00$ | $-3,420$ | $-7,800$ | 8,220 | 3,780 |
| 12 |  | $1,211,00$ | 0 | $-7,800$ | 12,000 |  |
| 13 |  | $1,214,30$ | 660 | $-7,140$ | 12,660 |  |
| 14 |  | $1,216.10$ | 360 | $-6,780$ | 13,020 |  |
| 15 |  | $1,223,00$ | 1,380 | $-5,400$ | 14,400 |  |
| 16 | $1,226,90$ |  | 780 | $-4,620$ | 15,180 |  |

5. (a) and (b)

Similar questions have been discussed in the lecture. Please go through the lectures of options and derivatives.

