NPTEL

INDUSTRIAL AND MANAGEMENT ENGINEERING DEPARTMENT, IIT KANPUR

QUANTITATIVE FINANCE

ASSIGNMENT-3(Solution) (2015 JULY-AUG ONLINE COURSE)

1. Please refer the sample solution provided for this question or refer this YouTube video whose link is given below:
   https://www.youtube.com/watch?v=FZyAXP4syD8

2.

   (a)

a. The naive method simply uses the demand for the current month as the forecast for the next month: $F_{t+1} = D_t$. So for February we would have $F_{\text{Feb.}} = D_{\text{Jan.}} = 120$. Similarly, $F_{\text{Nov.}} = D_{\text{Oct.}} = 90$. See the table below for the other months.

b. For a simple 3-month moving average, we take the average of the previous three months' demand as our forecast for next month: $F_{t+1} = \dfrac{D_t + D_{t-1} + D_{t-2}}{3}$. Since we need at least three months to compute the average, and we only have data beginning in January, April is the earliest month for which we can compute the forecast: $F_{\text{Apr.}} = \dfrac{D_{\text{Mar.}} + D_{\text{Feb.}} + D_{\text{Jan.}}}{3} = \dfrac{100 + 90 + 120}{3} = 103.3$. The forecasts for the other months are reported in the table below.

c. The 5-month moving average is similar to the 3-month moving average, except now we take the average of the previous five months' demand. We start with the forecast for June (since we need at least five months' worth of previous demand): $F_{\text{Jun.}} = \dfrac{D_{\text{May}} + D_{\text{Apr.}} + D_{\text{Mar.}} + D_{\text{Feb.}} + D_{\text{Jan.}}}{5} = \dfrac{110 + 75 + 100 + 90 + 120}{5} = 99.0$. The forecasts for the remaining months are computed similarly, and the values are reported in the table below.

d. Simple moving averages (like parts *b* and *c* above) place an equal weight on all of previous months. A weighted moving average allows us to put more weight on the more recent data. For a weighted 3-month moving average we have $F_{t+1} = w_1 D_t + w_2 D_{t-1} + w_3 D_{t-2}$. (Note that the weights should add up to 1.) Using the weights specified in the question, the forecast for April is computed as $F_{\text{Apr.}} = 0.5(D_{\text{Mar.}}) + 0.33(D_{\text{Feb.}}) + 0.17(D_{\text{Jan.}}) = 0.5(100) + 0.33(90) + 0.17(120) = 100.1$. Forecasts for May through November are reported in the table below.

		Forecast			
Month	Orders	Naive Method	3-Month Moving Avg.	5-Month Moving Avg.	3-Month Weighted Avg.
Jan.	120	—	—	—	—
Feb.	90	120	—	—	—
Mar.	100	90	—	—	—
Apr.	75	100	103.3	—	100.1
May	110	75	88.3	—	85.8
Jun.	50	110	95.0	99.0	96.8
Jul.	75	50	78.3	85.0	74.1
Aug.	130	75	78.3	82.0	72.7
Sep.	110	130	85.0	88.0	98.3
Oct.	90	110	105.0	95.0	110.7
Nov.	?	90	110.0	91.0	103.4

e. Mean absolute deviation is one measure of how close the forecast is to the actual demand. Recall that forecast error is simply $E_t = D_t - F_t$, and that the absolute deviation is simply the absolute value of error: $|E_t|$. For example, the error for the Naive Method for June is $E_{\text{Jun.}} = D_{\text{Jun.}} - F_{\text{Jun.}} = 50 - 110 = -60$. To compute the mean absolute deviation, take the absolute value of each error term, add them up, and divide by the number of terms: $\text{MAD} = \frac{\sum |E_t|}{n}$. (*Note:* You must take the absolute value of each error term *before* adding them up!) In this case, we compute the mean over five months. The error and MAD for the months June through October are reported below. In general, the forecast accuracy increases as more information is incorporated into the forecast.

		Error ($E_t = D_t - F_t$)			
Month	Orders	Naive Method	3-Month Moving Avg.	5-Month Moving Avg.	3-Month Weighted Avg.
Jun.	50	-60	-45.0	-49.0	-46.8
Jul.	75	25	-3.3	-10.0	0.9
Aug.	130	55	51.7	48.0	57.3
Sep.	110	-20	25.0	22.0	11.8
Oct.	90	-20	-15.0	-5.0	-20.7
	MAD	36.0	28.0	26.8	27.5

2.

(b)

a. The formula for exponential smoothing is: $F_{t+1} = F_t + \alpha(D_t - F_t)$. To determine the forecast for January, F_{13}, we need to know the forecast for December, F_{12}. This, in turn, requires us to know the forecast for November, F_{11}. So we need to go all the way back to the beginning and compute the forecast for each month. For Period 2, we have $F_2 = F_1 + \alpha(D_1 - F_1)$. But how do we get the forecast for Period 1? There are several ways to approach this, but we'll just use the demand for Period 1 as both *demand* and *forecast* for Period 1. Now we can write $F_2 = F_1 + \alpha(D_1 - F_1) = 37 + 0.3(37 - 37) = 37$. For Period 3 we have $F_3 = F_2 + \alpha(D_2 - F_2) = 37 + 0.3(40 - 37) = 37.9$. The forecasts for the other months are show in the table below. For Period 13 we have $F_{13} = F_{12} + \alpha(D_{12} - F_{12}) = 50.85 + 0.3(54 - 50.85) = 51.79$.

b. For $\alpha = 0.5$ we follow the same exact procedure as we did in part *a*. See the table below for the forecast values.

c. Incorporating a trend simply requires us to include a bit more information. The formula is: $F_{t+1} = A_t + T_t$ where $A_t = \alpha D_t + (1 - \alpha)(A_{t-1} + T_{t-1})$ and $T_t = \beta(A_t - A_{t-1}) + (1 - \beta)T_{t-1}$. Once again we need to go back to the beginning in order to find the necessary values to plug into the formula, and once again we need to make some assumptions about our initial values. For Period 2, we have $F_2 = A_1 + T_1$, so to get the process started, let $A_0 = 37$ and $T_0 = 0$. We can now compute A_1 and T_1 as follows: $A_1 = \alpha D_1 + (1 - \alpha)(A_0 + T_0) = 0.5(37) + (1 - 0.5)(37 + 0) = 37$, and $T_1 = \beta(A_1 - A_0) + (1 - \beta)T_0 = 0.3(37 - 37) + (1 - 0.3)(0) = 0$. Therefore, the forecast for Period 2 is $F_2 = A_1 + T_1 = 37 + 0 = 37$. For Period 3, we first compute A_2 and T_2 as follows: $A_2 = \alpha D_2 + (1 - \alpha)(A_1 + T_1) = 0.5(40) + (1 - 0.5)(37 + 0) = 38.5$, and $T_2 = \beta(A_2 - A_1) + (1 - \beta)T_1 = 0.3(38.5 - 37) + (1 - 0.3)(0) = 0.45$. The forecast for Period 3 is $F_3 = A_2 + T_2 = 38.5 + 0.45 = 38.95$. The forecasts for the remaining months are reported in the table below.

			Expon. Smooth.	Expon. Smooth.	Trend-Adjusted Expon. Smooth. ($\alpha = 0.5$, $\beta = 0.3$)		
Period	Month	Demand	$\alpha = 0.3$	$\alpha = 0.5$	A_t	T_t	F_t
1	Jan.	37	37.00	37.00	37.00	0.00	37.00
2	Feb.	40	37.00	37.00	38.50	0.45	37.00
3	Mar.	41	37.90	38.50	39.98	0.76	38.95
4	Apr.	37	38.83	39.75	38.87	0.20	40.73
5	May	45	38.28	38.38	42.03	1.09	39.06
6	Jun.	50	40.30	41.69	46.56	2.12	43.12
7	Jul.	43	43.21	45.84	45.84	1.27	48.68
8	Aug.	47	43.15	44.42	47.05	1.25	47.11
9	Sep.	56	44.30	45.71	52.15	2.41	48.31
10	Oct.	52	47.81	50.86	53.28	2.02	54.56
11	Nov.	55	49.07	51.43	55.15	1.98	55.30
12	Dec.	54	50.85	53.21	55.56	1.51	57.13
13	Jan.	?	51.79	53.61			57.07

e. To compute the mean square error, first compute the error for each period: $E_t = D_t - F_t$. Take that number and square it, then take the average over all periods: $\text{MSE} = \frac{\sum E_t^2}{n}$. (*Note:* You must square the error terms *before* adding them up!) Take the Exponential Smoothing method with $\alpha = 0.3$, for example. In the month of April, the error is $E_{\text{Apr.}} = D_{\text{Apr.}} - F_{\text{Apr.}} = 37 - 38.83 = -1.83$. We square this value, add it to the other squared error terms, and divide by 12 to get the mean. The error, squared error, and MSE for each of the methods are reported below. The trend-adjusted forecast, which incorporates the most information, has the highest accuracy (lowest MSE).

		Expon. Smooth. $\alpha = 0.3$		Expon. Smooth. $\alpha = 0.5$		Trend-Adj. $\alpha = 0.5, \beta = 0.3$	
Month	Demand	E_t	E_t^2	E_t	E_t^2	E_t	E_t^2
Jan.	37	0.00	0.00	0.00	0.00	0.00	0.00
Feb.	40	3.00	9.00	3.00	9.00	3.00	9.00
Mar.	41	3.10	9.61	2.50	6.25	2.05	4.20
Apr.	37	−1.83	3.35	−2.75	7.56	−3.73	13.93
May	45	6.72	45.14	6.63	43.89	5.94	35.24
Jun.	50	9.70	94.15	8.31	69.10	6.88	47.33
Jul.	43	−0.21	0.04	−2.84	8.09	−5.68	32.26
Aug.	47	3.85	14.86	2.58	6.65	−0.11	0.01
Sep.	56	11.70	136.85	10.29	105.86	7.69	59.20
Oct.	52	4.19	17.55	1.14	1.31	−2.56	6.55
Nov.	55	5.93	35.19	3.57	12.76	−0.30	0.09
Dec.	54	3.15	9.94	0.79	0.62	−3.13	9.78
		MSE	31.31		22.59		18.13

3.

(a) We think that there may be a relationship between class attendance and number of popcorn packets men sold. Data for the first six months are shown in the table. Forecast the number of popcorn packets men that will be sold in month 7 if monthly class attendance is forecast as 25000 people.

Month	Attendance (x) (,000)	Sales (y)	x^2	xy
1	8	1500	64	(8)(1500) = 12000
2	12	2200		
3	14	2700		
4	18	4200		
5	19	7800		
6	22	5400		
	$\sum x =$	$\sum y =$	$\sum x^2 =$	$\sum xy =$

$\bar{x} = \frac{\sum x}{n} =$ $\qquad$ $\bar{y} = \frac{\sum y}{n} =$

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} =$$

$$a = \bar{y} - b\bar{x} =$$

Thus, our regression equation is: $\hat{y}$ = + x

To calculate the forecast for month 7, we have: $\hat{y} =$

3 (b) Holt's Method

Month	***1999***	***2000***	***2001***	***Average Annual Demand***
1	1100	1300	1500	*(1100 + 1300 + 1500) / 3 = 1300*
2	1800	2000	2200	
3	2300	2500	2700	
4	3800	4000	4200	
5	4500	4700	4900	
6	5000	5200	5400	
7	5500	5700	*5900*	
8	4800	5000	*5200*	
9	3000	3200	*3400*	
10	2200	2400	*2600*	
11	1500	1700	*1900*	
12	1200	1400	*1600*	
				∑Average Monthly Demand =

Seasonal Index for January = 1300 / =

Forecast for January 2002 = $\frac{45000}{12} \times$ =

(Use excel to solve. It will be relatively easy)

4. (a)

Q1. (i) a) $y_t = \beta_0 + \beta_1 t + \varepsilon_t$

b) $y_3 = 250$

$S_3 = \alpha^* y_3 + (1-\alpha) S_2 = 217.8372$

$250\alpha + (1-\alpha)\, 185.6744 = 217.8372$

$\alpha = 0.5$

c) $S_0 = a_0(0) - \left(\frac{1-\alpha}{\alpha}\right) b_1(0)$

$= 17.2975 - \left(\frac{0.5}{0.5}\right) 34.6$

$= -17.3025$

$S_0^{(2)} = a_0(0) - 2\left(\frac{1-\alpha}{\alpha}\right) b_1(0)$

$= -51.9025$

d) $y_1 = 200$

$S_1 = 0.5(200) + 0.5(-17.3025) = 91.34875$

$S_1^{(2)} = 0.5(91.34875) + 0.5(-51.9025) = 19.723125$

$a_0(1) = 2S_1 - S_1^{(2)} = 162.974375$

$b_1(1) = \frac{0.5}{0.5}(91.34875 - 19.723125) = 71.625625$

$\therefore \hat{y}_{1+1}(1) = 162.974375 + 71.625625(1) = 234.6$

e) $S_9 = 0.5 y_9 + 0.5^* 412.8074 = 431.4037$

$\therefore y_9 = 450$

f) $\hat{y}_{10+2}(10) = a_0(10) + b_1(10)^* 2 = 626.40044$

$\hat{y}_{10+3}(10) = a_0(10) + b_1(10)^* 3 = 670.39996$

g)

Period	$y_t - \hat{y}_t$
2	45.4
3	-101.6256
4	-12.9756
5	-41.5692
6	-12.3253
7	7.067
8	-28.8517
9	-73.6184
10	69.5445

$\therefore SSE = 21293.15451$

Theil's U Stat $= \sqrt{\frac{21293.15451}{28071}}$

$= 0.87095$

(ii) a) $y_1 = 200$ p. 2

$a_0(1) = 0.2^* 200 + 0.8^* (19.2975 + 34.6) = 81.518$

$b_1(1) = 0.7^* (81.518 - 17.2975) + 0.3^* 34.6 = 55.33435$

$\hat{y}_{1+1}(1) = 136.85235$

$y_2 = 280$

$a_0(2) = 0.2^* 280 + 0.8 (81.518 + 55.33435) = 165.48188$

$b_1(2) = 0.7^* (165.48188 - 81.518) + 0.3^* 55.38435 = 75.375021$

$\hat{y}_{2+1}(2) = 240.856901$

b) By trial and error. Simulated forecasting is done on the historical data set using different values of α for one-parameter double exponential smoothing; and different combinations of α + β for HW's method. The α or combination of α + β which min. MSE is chosen.

c) For one-parameter exponential smoothing. Only one smoothing constant α is adopted. It's not good for forecasting medium to long term forecasting in general as $t\uparrow$.
Specific to HW's method, it lacks flexibility as α is applied to smooth both level + trend, while HW use α to smooth level + β to smooth trend, having more flexibilities.

Q2. a) - Data Series is given the name 'test 3'
- Data are input quarterly.
- from 98Q1 to 00Q4 data are input
- procedure x11 of SAS is adopted. By default, multiplicative decomposition is adopted.
- SAS will print out all output from x11.

b, $MA_{8.5} = 106.475$

P3

$MA_{9.5} = 106.85$

$\therefore CMA = 106.6625$,

$MA_{10.5} = 108.975$

$\therefore CMA_{10} = 107.9125$ //

c)

	Y_t	$CMA = tr_t \times cl_t$	$sn_t \times ir_t = Y_t / CMA$
Q3	100.2	107.45	0.93253
Q4	98.3	105.025	0.93597
Q1	103.4	103.013	1.00376
Q2	110.3	101.913	1.0823
Q3	99.9	102.05	0.9789
Q4	89.8	104.863	0.8564
Q1	113	106.6625	1.05942
Q2	123.2	107.9125	1.14167

			sn_t^*	sn_t
Q1	1.00376	1.05942	1.03159	1.03276
Q2	1.0823	1.14167	1.111985	1.11324
Q3	0.93253	0.9789	0.955715	0.956793
Q4	0.93597	0.8564	0.896185	0.8972

$\Sigma = 3.995475$

$\eta = 1.001132531$

$\therefore sn_2 = 1.11324$ //

$sn_3 = 0.956793$ //

d) Because seasonal factor only affects the season but not throughout the year. If there's no seasonal impact, $sn_1 = sn_2 = sn_3 = sn_4 = 1$.

Σsn_t should equal to 4 //

e) Expost forecast: forecast done when actual observations are known;

Ex-ante forecast: forecast done when actual observations not yet known.