NPTEL

INDUSTRIAL AND MANAGEMENT ENGINEERING DEPARTMENT, IIT KANPUR

QUANTITATIVE FINANCE

ASSIGNMENT-3(Solution) (2015 JULY-AUG ONLINE COURSE)

1. Please refer the sample solution provided for this question or refer this YouTube video whose link is given below:
   https://www.youtube.com/watch?v=FZyAXP4syD8

2.

   (a)

*a.* The naive method simply uses the demand for the current month as the forecast for the next month: $F_{t+1} = D_t$. So for February we would have $F_{\text{Feb.}} = D_{\text{Jan.}} = 120$. Similarly, $F_{\text{Nov.}} = D_{\text{Oct.}} = 90$. See the table below for the other months.

*b.* For a simple 3-month moving average, we take the average of the previous three months' demand as our forecast for next month: $F_{t+1} = \dfrac{D_t + D_{t-1} + D_{t-2}}{3}$. Since we need at least three months to compute the average, and we only have data beginning in January, April is the earliest month for which we can compute the forecast: $F_{\text{Apr.}} = \dfrac{D_{\text{Mar.}} + D_{\text{Feb.}} + D_{\text{Jan.}}}{3} = \dfrac{100 + 90 + 120}{3} = 103.3$. The forecasts for the other months are reported in the table below.

*c.* The 5-month moving average is similar to the 3-month moving average, except now we take the average of the previous five months' demand. We start with the forecast for June (since we need at least five months' worth of previous demand): $F_{\text{Jun.}} = \dfrac{D_{\text{May}} + D_{\text{Apr.}} + D_{\text{Mar.}} + D_{\text{Feb.}} + D_{\text{Jan.}}}{5} = \dfrac{110 + 75 + 100 + 90 + 120}{5} = 99.0$. The forecasts for the remaining months are computed similarly, and the values are reported in the table below.

*d.* Simple moving averages (like parts *b* and *c* above) place an equal weight on all of previous months. A weighted moving average allows us to put more weight on the more recent data. For a weighted 3-month moving average we have $F_{t+1} = w_1 D_t + w_2 D_{t-1} + w_3 D_{t-2}$. (Note that the weights should add up to 1.) Using the weights specified in the question, the forecast for April is computed as $F_{\text{Apr.}} = 0.5(D_{\text{Mar.}}) + 0.33(D_{\text{Feb.}}) + 0.17(D_{\text{Jan.}}) = 0.5(100) + 0.33(90) + 0.17(120) = 100.1$. Forecasts for May through November are reported in the table below.

| | | Forecast | | | |
|---|---|---|---|---|---|
| | | Naive | 3-Month | 5-Month | 3-Month |
| Month | Orders | Method | Moving Avg. | Moving Avg. | Weighted Avg. |
| Jan. | 120 | — | — | — | — |
| Feb. | 90 | 120 | — | — | — |
| Mar. | 100 | 90 | — | — | — |
| Apr. | 75 | 100 | 103.3 | — | 100.1 |
| May | 110 | 75 | 88.3 | — | 85.8 |
| Jun. | 50 | 110 | 95.0 | 99.0 | 96.8 |
| Jul. | 75 | 50 | 78.3 | 85.0 | 74.1 |
| Aug. | 130 | 75 | 78.3 | 82.0 | 72.7 |
| Sep. | 110 | 130 | 85.0 | 88.0 | 98.3 |
| Oct. | 90 | 110 | 105.0 | 95.0 | 110.7 |
| Nov. | ? | 90 | 110.0 | 91.0 | 103.4 |

*c.* Mean absolute deviation is one measure of how close the forecast is to the actual demand. Recall that forecast error is simply $E_t = D_t - F_t$, and that the absolute deviation is simply the absolute value of error: $|E_t|$. For example, the error for the Naive Method for June is $E_{\text{Jun.}} = D_{\text{Jun.}} - F_{\text{Jun.}} = 50 - 110 = -60$. To compute the mean absolute deviation, take the absolute value of each error term, add them up, and divide by the number of terms: $\text{MAD} = \frac{\sum |E_t|}{n}$. (*Note:* You must take the absolute value of each error term *before* adding them up!) In this case, we compute the mean over five months. The error and MAD for the months June through October are reported below. In general, the forecast accuracy increases as more information is incorporated into the forecast.

| | | Error ($E_t = D_t - F_t$) | | | |
|---|---|---|---|---|---|
| | | Naive | 3-Month | 5-Month | 3-Month |
| Month | Orders | Method | Moving Avg. | Moving Avg. | Weighted Avg. |
| Jun. | 50 | −60 | −45.0 | −49.0 | −46.8 |
| Jul. | 75 | 25 | −3.3 | −10.0 | 0.9 |
| Aug. | 130 | 55 | 51.7 | 48.0 | 57.3 |
| Sep. | 110 | −20 | 25.0 | 22.0 | 11.8 |
| Oct. | 90 | −20 | −15.0 | −5.0 | −20.7 |
| | **MAD** | 36.0 | 28.0 | 26.8 | 27.5 |

2.

(b)

*a*. The formula for exponential smoothing is: $F_{t+1} = F_t + \alpha(D_t - F_t)$. To determine the forecast for January, $F_{13}$, we need to know the forecast for December, $F_{12}$. This, in turn, requires us to know the forecast for November, $F_{11}$. So we need to go all the way back to the beginning and compute the forecast for each month. For Period 2, we have $F_2 = F_1 + \alpha(D_1 - F_1)$. But how do we get the forecast for Period 1? There are several ways to approach this, but we'll just use the demand for Period 1 as both *demand* and *forecast* for Period 1. Now we can write $F_2 = F_1 + \alpha(D_1 - F_1) = 37 + 0.3(37 - 37) = 37$. For Period 3 we have $F_3 = F_2 + \alpha(D_2 - F_2) = 37 + 0.3(40 - 37) = 37.9$. The forecasts for the other months are show in the table below. For Period 13 we have $F_{13} = F_{12} + \alpha(D_{12} - F_{12}) = 50.85 + 0.3(54 - 50.85) = 51.79$.

*b*. For $\alpha = 0.5$ we follow the same exact procedure as we did in part *a*. See the table below for the forecast values.

*c*. Incorporating a trend simply requires us to include a bit more information. The formula is: $F_{t+1} = A_t + T_t$ where $A_t = \alpha D_t + (1-\alpha)(A_{t-1} + T_{t-1})$ and $T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$. Once again we need to go back to the beginning in order to find the necessary values to plug into the formula, and once again we need to make some assumptions about our initial values. For Period 2, we have $F_2 = A_1 + T_1$, so to get the process started, let $A_0 = 37$ and $T_0 = 0$. We can now compute $A_1$ and $T_1$ as follows: $A_1 = \alpha D_1 + (1-\alpha)(A_0 + T_0) = 0.5(37) + (1-0.5)(37+0) = 37$, and $T_1 = \beta(A_1 - A_0) + (1-\beta)T_0 = 0.3(37 - 37) + (1 - 0.3)(0) = 0$. Therefore, the forecast for Period 2 is $F_2 = A_1 + T_1 = 37 + 0 = 37$. For Period 3, we first compute $A_2$ and $T_2$ as follows: $A_2 = \alpha D_2 + (1-\alpha)(A_1 + T_1) = 0.5(40) + (1-0.5)(37+0) = 38.5$, and $T_2 = \beta(A_2 - A_1) + (1-\beta)T_1 = 0.3(38.5 - 37) + (1 - 0.3)(0) = 0.45$. The forecast for Period 3 is $F_3 = A_2 + T_2 = 38.5 + 0.45 = 38.95$. The forecasts for the remaining months are reported in the table below.

| | | | Expon. | Expon. | Trend-Adjusted Expon. | | |
|---|---|---|---|---|---|---|---|
| | | | Smooth. | Smooth. | Smooth. ($\alpha = 0.5$, $\beta = 0.3$) | | |
| Period | Month | Demand | $\alpha = 0.3$ | $\alpha = 0.5$ | $A_t$ | $T_t$ | $F_t$ |
| 1 | Jan. | 37 | 37.00 | 37.00 | 37.00 | 0.00 | 37.00 |
| 2 | Feb. | 40 | 37.00 | 37.00 | 38.50 | 0.45 | 37.00 |
| 3 | Mar. | 41 | 37.90 | 38.50 | 39.98 | 0.76 | 38.95 |
| 4 | Apr. | 37 | 38.83 | 39.75 | 38.87 | 0.20 | 40.73 |
| 5 | May | 45 | 38.28 | 38.38 | 42.03 | 1.09 | 39.06 |
| 6 | Jun. | 50 | 40.30 | 41.69 | 46.56 | 2.12 | 43.12 |
| 7 | Jul. | 43 | 43.21 | 45.84 | 45.84 | 1.27 | 48.68 |
| 8 | Aug. | 47 | 43.15 | 44.42 | 47.05 | 1.25 | 47.11 |
| 9 | Sep. | 56 | 44.30 | 45.71 | 52.15 | 2.41 | 48.31 |
| 10 | Oct. | 52 | 47.81 | 50.86 | 53.28 | 2.02 | 54.56 |
| 11 | Nov. | 55 | 49.07 | 51.43 | 55.15 | 1.98 | 55.30 |
| 12 | Dec. | 54 | 50.85 | 53.21 | 55.56 | 1.51 | 57.13 |
| 13 | Jan. | ? | 51.79 | 53.61 | | | 57.07 |

e. To compute the mean square error, first compute the error for each period: $E_t = D_t - F_t$. Take that number and square it, then take the average over all periods: $\text{MSE} = \frac{\sum E_t^2}{n}$. (*Note:* You must square the error terms *before* adding them up!) Take the Exponential Smoothing method with $\alpha = 0.3$, for example. In the month of April, the error is $E_{\text{Apr.}} = D_{\text{Apr.}} - F_{\text{Apr.}} = 37 - 38.83 = -1.83$. We square this value, add it to the other squared error terms, and divide by 12 to get the mean. The error, squared error, and MSE for each of the methods are reported below. The trend-adjusted forecast, which incorporates the most information, has the highest accuracy (lowest MSE).

| | | Expon. Smooth. $\alpha = 0.3$ | | Expon. Smooth. $\alpha = 0.5$ | | Trend-Adj. $\alpha = 0.5, \beta = 0.3$ | |
|---|---|---|---|---|---|---|---|
| Month | Demand | $E_t$ | $E_t^2$ | $E_t$ | $E_t^2$ | $E_t$ | $E_t^2$ |
| Jan. | 37 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Feb. | 40 | 3.00 | 9.00 | 3.00 | 9.00 | 3.00 | 9.00 |
| Mar. | 41 | 3.10 | 9.61 | 2.50 | 6.25 | 2.05 | 4.20 |
| Apr. | 37 | −1.83 | 3.35 | −2.75 | 7.56 | −3.73 | 13.93 |
| May | 45 | 6.72 | 45.14 | 6.63 | 43.89 | 5.94 | 35.24 |
| Jun. | 50 | 9.70 | 94.15 | 8.31 | 69.10 | 6.88 | 47.33 |
| Jul. | 43 | −0.21 | 0.04 | −2.84 | 8.09 | −5.68 | 32.26 |
| Aug. | 47 | 3.85 | 14.86 | 2.58 | 6.65 | −0.11 | 0.01 |
| Sep. | 56 | 11.70 | 136.85 | 10.29 | 105.86 | 7.69 | 59.20 |
| Oct. | 52 | 4.19 | 17.55 | 1.14 | 1.31 | −2.56 | 6.55 |
| Nov. | 55 | 5.93 | 35.19 | 3.57 | 12.76 | −0.30 | 0.09 |
| Dec. | 54 | 3.15 | 9.94 | 0.79 | 0.62 | −3.13 | 9.78 |
| | | **MSE** | 31.31 | | 22.59 | | 18.13 |

3.

(a) We think that there may be a relationship between class attendance and number of popcorn packets men sold. Data for the first six months are shown in the table. Forecast the number of popcorn packets men that will be sold in month 7 if monthly class attendance is forecast as 25000 people.

| Month | Attendance (x) (,000) | Sales (y) | $x^2$ | xy |
|---|---|---|---|---|
| 1 | 8 | 1500 | 64 | (8)(1500) = 12000 |
| 2 | 12 | 2200 | | |
| 3 | 14 | 2700 | | |
| 4 | 18 | 4200 | | |
| 5 | 19 | 7800 | | |
| 6 | 22 | 5400 | | |
| | $\sum x =$ | $\sum y =$ | $\sum x^2 =$ | $\sum xy =$ |

$\bar{x} = \frac{\sum x}{n} =$ $\bar{y} = \frac{\sum y}{n} =$

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} =$$

$$a = \bar{y} - b\bar{x} =$$

Thus, our regression equation is: $\hat{y}$ = + x

To calculate the forecast for month 7, we have: $\hat{y} =$

3 (b) Holt's Method

| ***Month*** | ***1999*** | ***2000*** | ***2001*** | ***Average Annual Demand*** |
|---|---|---|---|---|
| *1* | 1100 | 1300 | 1500 | *(1100 + 1300 + 1500) / 3 = 1300* |
| *2* | 1800 | 2000 | 2200 | |
| *3* | 2300 | 2500 | 2700 | |
| *4* | 3800 | 4000 | 4200 | |
| *5* | 4500 | 4700 | 4900 | |
| *6* | 5000 | 5200 | 5400 | |
| *7* | 5500 | 5700 | *5900* | |
| *8* | 4800 | 5000 | *5200* | |
| *9* | 3000 | 3200 | *3400* | |
| *10* | 2200 | 2400 | *2600* | |
| *11* | 1500 | 1700 | *1900* | |
| *12* | 1200 | 1400 | *1600* | |
| | | | | *$\sum$ Average Monthly Demand =* |

*Seasonal Index for January = 1300 /* =

*Forecast for January 2002 =* $\frac{45000}{12} \times$ =

*(Use excel to solve. It will be relatively easy)*

4. (a)

Q1. (i) a) $y_t = \beta_0 + \beta_1 t + \varepsilon_t$

b) $y_3 = 250$

$S_3 = \alpha^* y_3 + (1-\alpha) S_2 = 217.8372$

$250\alpha + (1-\alpha)\, 185.6744 = 217.8372$

$\alpha = 0.5$

c) $S_0 = a_0(0) - \left(\frac{1-\alpha}{\alpha}\right) b_1(0)$

$= 17.2975 - \left(\frac{0.5}{0.5}\right) 34.6$

$= -17.3025$

$S_0^{(2)} = a_0(0) - 2\left(\frac{1-\alpha}{\alpha}\right) b_1(0)$

$= -51.9025$

d) $y_1 = 200$

$S_1 = 0.5(200) + 0.5(-17.3025) = 91.34875$

$S_1^{(2)} = 0.5(91.34875) + 0.5(-51.9025) = 19.723125$

$a_0(1) = 2S_1 - S_1^{(2)} = 162.974375$

$b_1(1) = \frac{0.5}{0.5}(91.34875 - 19.723125) = 71.625625$

$\therefore \hat{y}_{1+1}(1) = 162.974375 + 71.625625(1) = 234.6$

e) $S_9 = 0.5 y_9 + 0.5 * 412.8074 = 431.4037$

$\therefore y_9 = 450$

f) $\hat{y}_{10+2}(10) = a_0(10) + b_1(10)^* 2 = 626.40044$

$\hat{y}_{10+3}(10) = a_0(10) + b_1(10)^* 3 = 670.79996$

g)

| Period | $y_t - \hat{y}_t$ |
|---|---|
| 2 | 45.4 |
| 3 | -101.6256 |
| 4 | -12.9756 |
| 5 | -41.5692 |
| 6 | -12.3253 |
| 7 | 7.067 |
| 8 | -28.8517 |
| 9 | -33.6184 |
| 10 | 69.5945 |

$\therefore SSE = 21293.15451$

Theil'u Stat $= \sqrt{\frac{21293.15451}{28071}}$

$= 0.87095$

(ii) a) $y_1 = 200$ 

$a_0(1) = 0.2^* 200 + 0.8^* (19.2975 + 34.6) = 81.518$

$b_1(1) = 0.7^* (81.518 - 17.2975) + 0.3^* 34.6 = 55.33435$

$\hat{y}_{1+1}(1) = 136.85235$

$y_2 = 280$

$a_0(2) = 0.2^* 280 + 0.8 (81.518 + 55.33435) = 165.48188$

$b_1(2) = 0.7^* (165.48188 - 81.518) + 0.3^* 55.33435 = 75.375021$

$\hat{y}_{2+1}(2) = 240.856901$

b) By trial and error. Simulated forecasting is done on the historical data set using different values of $\alpha$ for one-parameter double exponential smoothing; and different combinations of $\alpha$ + $\beta$ for HW's method. The $\alpha$ or combination of $\alpha$ + $\beta$ which min. MSE is chosen

c) For one-parameter exponential smoothing. Only one smoothing constant $\alpha$ is adopted. It's not good for forecasting medium to long term forecasting in general as t↑.
Specific to HW's method, it lacks flexibility as $\alpha$ is applied to smooth both level + trend, while HW use $\alpha$ to smooth level + $\beta$ to smooth trend, having more flexibilities.

Q2. a) - Data Series is given the name 'test 3'
- Data are input quarterly.
- from 98Q1 to 00Q4 data are input
- procedure x11 of SAS is adopted. By default, multiplicative decomposition is adopted.
- SAS will print out all output from x11.

P3

b, $MA_{8.5} = 106.475$

$MA_{9.5} = 106.85$

$\therefore CMA = 106.6625$,

$MA_{10.5} = 108.975$

$\therefore CMA_{10} = 107.9125$ //

c)

| | $Y_t$ | $CMA = tr_t \times Cl_t$ | $Sn_t \times ir_t = Y_t / CMA$ |
|---|---|---|---|
| Q3 | 100.2 | 107.45 | 0.93253 |
| Q4 | 98.3 | 105.025 | 0.93597 |
| Q1 | 103.4 | 103.013 | 1.00376 |
| Q2 | 110.3 | 101.913 | 1.0823 |
| Q3 | 99.9 | 102.05 | 0.9789 |
| Q4 | 89.8 | 104.863 | 0.8564 |
| Q1 | 113 | 106.6625 | 1.05942 |
| Q2 | 123.2 | 107.9125 | 1.14167 |

| | | | $Sn_t^*$ | $Sn_t$ |
|---|---|---|---|---|
| Q1 | 1.00376 | 1.05942 | 1.03159 | 1.03276 |
| Q2 | 1.0823 | 1.14167 | 1.111985 | 1.11324 |
| Q3 | 0.93253 | 0.9789 | 0.955715 | 0.956797 |
| Q4 | 0.93597 | 0.8564 | 0.896185 | 0.8972 |

$\Sigma = 3.995475$

$\eta = 1.001132531$

$\therefore Sn_2 = 1.11324$ //

$Sn_3 = 0.956797$ //

d) Because seasonal factor only affects the season but not throughout the year. If there's no seasonal impact, $Sn_1 = Sn_2 = Sn_3 = Sn_4 = 1$.

$\Sigma Sn_t$ should equal to 4 //

e) Ex-post forecast: forecast done when actual observations are known;

Ex-ante forecast: forecast done when actual observations not yet known.