NPTEL

INDUSTRIAL AND MANAGEMENT ENGINEERING DEPARTMENT, IIT KANPUR

QUANTITATIVE FINANCE

ASSIGNMENT-2 (2015 JULY-AUG ONLINE COURSE)

NOTE THE FOLLOWING

1) There are six questions and you are required to answer all of them.

2) Deadline for submission is Wednesday; 22nd July, 2015

3) The total marks is 50.

4) To get full credit do your calculations carefully.

Question # 1:

(a) Derive the equation for the efficient portfolio frontier and its asymptotes **(without risk free asset)** and show each step of calculation involving matrix algebra with proper explanation. Refer study material # 3. You have to show complete calculations for $\lambda$ & $\gamma$ that are missing from it. Marks will be granted for logical reasoning and clear explanation of each step. Also, derive an algebraic expression (MATRIX form) for the global minimum variance portfolio, the corresponding return and its corresponding weights.

(b) Derive the equation for the efficient portfolio frontier and its asymptotes **(with a risk free asset)** and show each step of calculation involving matrix algebra with proper explanation. Refer study material # 3. You have to show complete calculations for $\lambda$ & $\gamma$ that are missing from it. Marks will be granted for logical reasoning and clear explanation of each step.

Question # 2

(a) Consider the mean-variance analysis covered in this week's lecture where there are n risky assets whose returns are jointly normally distributed. Assume that investors differ with regard to their (concave) utility functions and their initial wealth. Also assume that investors can lend at the risk-free rate, $R_f < R_{mv}$, but investors are restricted from risk-free borrowing; that is, no risk-free borrowing is permitted.

i. Given this risk-free borrowing restriction, graphically show the efficient frontier for these investors in expected portfolio return-standard deviation space ($R_p$, $\sigma_p$ )

ii. Explain why only three portfolios are needed to construct this efficient frontier, and locate these three portfolios on your graph. (Note that these portfolios may not be unique.)

iii. At least one of these portfolios will sometimes need to be sold short to generate the entire efficient frontier. Which portfolio(s) is it (label it on the graph) and in what range(s) of the efficient frontier will it be sold short? Explain.

(b) We have the average rate of returns for a particular stock ABC, the market index and the T-Bill for ten consecutive time periods.

| Time | ABC | Market Index | T-Bill |
|---|---|---|---|
| 1 | 11 | 6 | 7 |
| 2 | 12 | 13 | 8 |
| 3 | 13 | 14 | 6 |
| 4 | 12 | 10 | 6 |
| 5 | 11 | 10 | 8 |
| 6 | -9 | -5 | 7 |
| 7 | 9 | 13 | 7 |
| 8 | 12 | 13 | 8 |
| 9 | 10 | 9 | 8 |
| 10 | 11 | 11 | 7 |
| | | | |
| **Average** | 9.2 | 9.4 | 7.2 |
| **SD** | 6.2 | 5.3 | 0.8 |

$\rho_{\text{ABC, Market Index}} = 0.914$, $\beta_{\text{ABC}} = 1.1$

Find the J values for each of the ten years and then the S value for these ten years taken as one time window. Can you comment about the characteristics of the stock ABC and the market index?

Question # 3

(a) Taking the index of **one** company (01-01-2005 – 30-06-2006) design a multi index model. You are free to choose the variables which you think are appropriate. This is just a rudimentary exercise to check the working principle of the multi index model. This work may not yield you an exact answer but would give you an idea about the working principle of multi index model.

(b) Consider there is no risk free interest rate. Prove that other assets can be priced according to $\overline{R}_i - \overline{R}_z = \beta_{i,M}\left(\overline{R}_M - \overline{R}_z\right)$, where z is some base portfolio/index. What is the beta of this portfolio/index?

Question # 4

(a) While calculating the efficient frontier for the case where we have a risk-free security (rate of return being $r_f$) along with a portfolio consisting of ″n″ number of risky assets, under the assumptions that:

(i) Short sale is allowed and

(ii) risk less lending and borrowing is allowed,

We put $\frac{\partial \tan\theta}{\partial w_k} = 0$, for k = 1, 2,….., n and find the values of $w_k$. But now if you consider that (i) risk less lending and borrowing is possible but (ii) short sale is not allowed, how would you formulate the problem to find out the optimum weights $w_i$′s of the ″n″ number of risky assets in the portfolio? You are given the following information, such that you are required to form the optimal portfolio consisting of assets A1 to A5 in some proportions. Assume SS is allowed.

(b) Using the concept of geometric mean return, which combination of assets, A, B and C from the three situations/cases would you select?

a. Case I: $R_A$ = 5, P(A) = 0.25; $R_B$ = 7, P = 0.25; $R_C$ = 10, P(C) = 0.50
b. Case II: $R_A$ = 7, P(A) = 1/3; $R_B$ = 7, P = 1/3; $R_C$ = 9, P(C) = 1/3
c. Case III: $R_A$ = 10, P(A) = 0.50; $R_B$ = 7, P = 0.30; $R_C$ = 4, P(C) = 0.20

Question # 5

a) What are the assumptions for CAPM? Explain at-least 5 such assumptions in your own words.
b) What is an efficient frontier? How do the two important properties of non-satiation and risk aversion help you to delineate the efficient frontier?
c) State the concept of one fund theorem and two fund’s theorem. (Show diagrams also)
d) State whether the following statements are true or false and give adequate and valid reasons (not necessarily mathematical) for your corresponding answers:
   i. Quadratic utility function would imply that returns are normally distributed.
   ii. Inefficiency in the price of any stock can be measured by the Jensen index.
   iii. Inefficiency in the market can be measured by the Sharpe ratio.
   iv. The properties of non-satiation and risk aversion lead us to understand the shape and form of the efficient frontier.