NPTEL

INDUSTRIAL AND MANAGEMENT ENGINEERING DEPARTMENT, IIT KANPUR

QUANTITATIVE FINANCE

ASSIGNMENT-1 (2015 JULY-AUG ONLINE COURSE)

1. (a) $U(W) = e^{aW} + b * W + c$ and we are required to find the properties of this utility function Now $U'(W) = ae^{aW} + b$ and $U''(W) = a^2 e^{aW}$.

   As $U'(W) > 0$ and $U''(W) > 0$, hence the utility function has the two fundamental property of (i) <u>non-satiation</u> and (ii) <u>risk averseness</u>. Now let us find <u>absolute risk aversion</u> and <u>relative risk aversion</u> properties of this particular utility function.

   $A\ W = -\frac{U''(W)}{U'(W)} = -\frac{a^2 e^{aW}}{ae^{aW}+b}$ and $R\ W = -W\ \frac{U''(W)}{U'(W)} = -W\ \frac{a^2 e^{aW}}{ae^{aW}+b}$

   Now from the two equations we easily see that:

   i. $A'(W) < 0$, which implies <u>decreasing absolute risk aversion</u> property, i.e., as the amount of wealth (W) increases the amount held in risky assets also increases.

   ii. $R'(W) < 0$, which implies <u>decreasing relative risk aversion</u> property, i.e., as the amount of wealth (W) increases the % held in risky assets also increases.

2.

   (a) We have $u'\ w = \frac{1}{2}w^{-\frac{1}{2}}$, so $u''\ w = -\frac{1}{4}w^{-\frac{3}{2}}$. As we will see below, $u''\ w < 0$ indicates that the individual is risk-averse.

   (b) The expected amount of money he will lose is:

   $$.25\quad Rs.0.8k + .75\quad 0 = Rs.0.2k$$

   His expected wealth is:

   $$.25\quad Rs.0.2k + .75\quad Rs.1k = Rs.0.8k$$

   His expected utility is

   $$(.25) \cdot u(Rs.0.2k) + (.75) \cdot u(Rs.1k)$$

   (c) His certainty equivalent wealth is the certain wealth w_{CE} that gives him the same expected utility as the uncertain certain he starts out it, i.e., the certain wealth w_{CE} that gives him an expected utility of 861.80. Solving $u(w_{CE})$ for w_{CE} gives us w_{CE}.

   (d) The maximum amount he would pay for full insurance is his initial wealth minus his certainty equivalent wealth:

   (e) The second derivative is now $u''\ w = -\frac{3}{4}w^{-\frac{3}{2}}$, which is of the same sign as before but three times larger in magnitude. His expected loss and expected wealth are unchanged at $Rs.0.2k$ and $Rs.0.8k$, respectively. His expected utility is now $(.25) \cdot u(Rs.0.2k) + (.75) \cdot u(Rs.1k)$. Calculate rest others.

3. (a) Please go through Funds Separation theorem and solve the question using spreadsheet.

(b)

Given

$\bar{r}_A = 0.15 = 15\%$ $\quad \sigma_A = 0.05 = 5\%$

$\bar{r}_B = 0.25 = 25\%$ $\quad \sigma_b = 0.15 = 15\%$

Let α be weight of r_A

$$\bar{R}_P = \alpha \bar{r}_A + (1-\alpha)\bar{r}_B$$

$$\sigma_P^2 = \alpha^2 \sigma_A^2 + (1-\alpha)^2 \sigma_b^2 + 2\alpha(1-\alpha)\sigma_A\sigma_B\rho_{AB}(-0.5)$$

$$R_P = 15\alpha + 25 - 25\alpha$$

$$\alpha = \frac{25 - R_P}{10}$$

$$\sigma_P^2 = 25\cdot\alpha^2 + (1-\alpha)^2\cdot 225 + 2\cdot\alpha(1-\alpha)(-0.5)(5\times 15)$$

$$= 25\alpha^2 + (1-\alpha)^2\, 225 - 75(\alpha)(1-\alpha)$$

Substituting value of α from R_P

$$\sigma_P^2 = 25\left(\frac{25-R_P}{10}\right)^2 + \left(1 - \frac{25-R_P}{10}\right)^2\cdot 225 - 75\left(\frac{25-R_P}{10}\right)\left(1 - \frac{25-R_P}{10}\right)$$

$$\sigma_p^2 = \left(\frac{25 - R_p}{2 \cdot 10}\right)^2 \cdot 25 + 9 \cdot 225 \cdot \left(\frac{R_p - 15}{2 \cdot 10}\right)^2 - \frac{3}{75} \cdot \left(\frac{R_p - 15}{2 \cdot 10}\right)\left(\frac{25 - R_p}{10 \cdot 2}\right)$$

$$4\sigma_p^2 = \underline{R_p^2} + 625 - 50R_p + \underline{9R_p^2} + 2025 - 270\,R_p$$

$$+ \underline{3R_p^2} + 1125 - 120R_p$$

$$4\sigma_p^2 = 13R_p^2 - 440\,R_p + 3775\,.$$

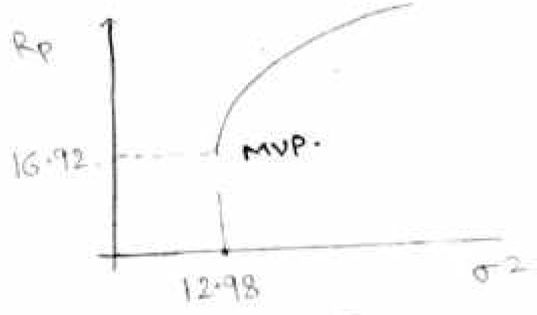
$$\frac{4}{13}\sigma^2 = R_p^2 - 33.84R_p + 290.38$$

$$= (R_p - 16.92)^2 + 3.994.$$

$$(R_p - 16.92)^2 = \frac{4}{13}\sigma^2 - 3.994$$

$$= \frac{4}{13}\left(\sigma^2 - 12.98\right)$$

So efficient frontier looks like a parabola. equation.



So MPV corresponds to $(R_p, \sigma^2) \equiv (16.92, 12.98)$

$\sigma = 3.6\%$

for return at 20% we have $\sigma^2 = 43.81\%$

$\sigma = 6.62\%$

4 and 5

Solve using Spreadsheet

6. (a) (iii)

(b) False

(c) True

(d) If one of the w's is negative and your normalization scheme preserves signs, then surely after normalization one of them would exceed one.

(e) No, it would be treated as a risky asset