Unit 7 - Week 6: Properties of measures of Information-1

Weekly Assignment 6

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-10-28, 23:59 IST.

Note: All the questions below may have multiple correct answers. There is no negative making for wrong choices made. However, partial marking is considered only when none of the marked choices is wrong.
Unless explicitly mentioned, assume that probability distributions are defined over a finite alphabet.

1) Let P and Q be two distributions over a finite alphabet $\mathcal{X}$. Which of the following are true? *1 point*

- $\sum_{x\in\mathcal{X}} P(x)\log\frac{1}{Q(x)} \leq H(P)$
- $\sum_{x\in\mathcal{X}} P(x)\log\frac{1}{Q(x)} \geq H(P)$
- If P is a uniform distribution and $Q=(1,0,\ldots,0)$, then $D(P||Q)=\log|\mathcal{X}|$.
- If P is a uniform distribution and $Q=(1,0,\ldots,0)$, then $D(P||Q)>\log|\mathcal{X}|$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\sum_{x\in\mathcal{X}} P(x)\log\frac{1}{Q(x)} \geq H(P)$
If P is a uniform distribution and $Q=(1,0,\ldots,0)$, then $D(P||Q)>\log|\mathcal{X}|$.

2) Let X, Y be random variables with joint distribution $P_{X,Y}$. Which of the following are true? *1 point*

- $I(X\wedge Y)=\mathbb{E}_{P_X}[D(P_{Y|X}||P_Y)]$
- $I(X\wedge Y)=\mathbb{E}_{P_Y}[D(P_{X|Y}||P_X)]$
- $I(X\wedge Y)=D(P_{X,Y}||P_XP_Y)$
- $I(Y\wedge X)=D(P_XP_Y||P_{X,Y})$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$I(X\wedge Y)=\mathbb{E}_{P_X}[D(P_{Y|X}||P_Y)]$
$I(X\wedge Y)=\mathbb{E}_{P_Y}[D(P_{X|Y}||P_X)]$
$I(X\wedge Y)=D(P_{X,Y}||P_XP_Y)$

3) Suppose X is a positive-valued random variable such that $\mathbb{E}[X]=2$. Let $f(X)=X\log X$ and $g(X)=\log X$. Which of the following are true? *1 point*

- $\mathbb{E}[f(X)]\leq 2$
- $\mathbb{E}[f(X)]\geq 2$
- $\mathbb{E}[f(X)]\geq \mathbb{E}[g(X)]+1$
- $\mathbb{E}[f(X)]\leq \mathbb{E}[g(X)]+1$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mathbb{E}[f(X)]\geq 2$
$\mathbb{E}[f(X)]\geq \mathbb{E}[g(X)]+1$

4) Let X be a random variable taking values in an alphabet $\mathcal{X}$. Let $Y=f(X)$, where $f:\mathcal{X}\to\mathcal{Y}$ is a deterministic function. Which of the following are true? *1 point*

- $H(X)\leq H(X,Y)$
- $H(X)\geq H(X,Y)$
- $H(X)=H(Y)$
- $H(Y|X)=0$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$H(X)\leq H(X,Y)$
$H(X)\geq H(X,Y)$
$H(Y|X)=0$

5) Let P_1 and P_2 be probability distributions over alphabet $\mathcal{X}$ such that $H(P_1)=H(P_2)=2$. Let $P=\theta P_1+(1-\theta)P_2$, where $\theta\in[0,1]$. Which of the following are true? *1 point*

- $H(P)<2$ for $\theta\in(0,1/2)$
- P may not be a valid probability distribution, so $H(P)$ may not be well-defined.
- $H(P)\geq 2$ for every $\theta\in[0,1]$
- $H(P)=2$ for every $\theta\in[0,1]$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$H(P)\geq 2$ *for every* $\theta\in[0,1]$

6) Which of the following are true? *1 point*

- $f(x)=x$ for $x>0$ is a concave function.
- $f(x)=\log x$ for $x>0$ is a concave function
- $f(x)=|x|$ for $x\in\mathbb{R}$ is a concave function
- $f(x)=\max\{x, x\log x\}$ for $x>0$ is a concave function

No, the answer is incorrect.
Score: 0
Accepted Answers:
$f(x)=x$ for $x>0$ is a concave function.
$f(x)=\log x$ for $x>0$ is a concave function

7) Let X, Y, Z be random variables with joint distribution $P_{X,Y,Z}$. Which of the following are true? *1 point*

- $H(X,Y)=H(X)+H(X|Y)$
- $H(X,Y)\leq H(X)+H(Y)$
- $H(X,Y)\geq H(X|Z)+H(Y|X)$
- $H(X,Y)\leq H(X|Z)+H(Y|X)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$H(X,Y)\leq H(X)+H(Y)$
$H(X,Y)\geq H(X|Z)+H(Y|X)$

8) Let X, Y, Z be random variables with joint probability distribution $P_{X,Y,Z}$. Which of the following are true? *1 point*

- $I(X\wedge Y)\leq I(X\wedge Y,Z)$
- $I(X\wedge Y|Z)\geq I(X\wedge Y)$
- $I(X\wedge Y|Z)\leq I(X\wedge Y)$
- $I(X\wedge Y|Z)\leq I(X\wedge Y,Z)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$I(X\wedge Y)\leq I(X\wedge Y,Z)$
$I(X\wedge Y|Z)\leq I(X\wedge Y,Z)$

9) Let X, Y be independent $\mathrm{Ber}(1/2)$ random variables. Define $Z=X\oplus Y$, where $\oplus$ is the XOR operator. Which of the following are true? *1 point*

- $I(X\wedge Y)=0$
- $I(X\wedge Z)=0$
- $I(X\wedge Y|Z)=0$
- $I(X\wedge Y|Z)=1$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$I(X\wedge Y)=0$
$I(X\wedge Z)=0$
$I(X\wedge Y|Z)=1$

10) Let X, Y, Z be random variables that form a Markov chain $X-Y-Z$. In the lectures, we saw the data processing inequality, $I(X\wedge Z)\leq I(X\wedge Y)$. When does this inequality become a strict equality? *1 point*

- If and only if $I(X\wedge Z)=I(X\wedge Y,Z)$
- If and only if $I(X\wedge Y|Z)=0$
- If and only if X, Y, Z are mutually independent
- If and only if X, Y, Z form a Markov chain $X-Z-Y$

No, the answer is incorrect.
Score: 0
Accepted Answers:
If and only if $I(X\wedge Z)=I(X\wedge Y,Z)$
If and only if $I(X\wedge Y|Z)=0$
If and only if X, Y, Z *form a Markov chain* $X-Z-Y$