

Unit 7 - Week 6: Properties of measures of Information-1

Course outline
How does an NPTEL online course work?
Week 1: Information and probabilistic modeling
Week 2: Uncertainty, compression, and entropy
Week 3: Randomness and entropy
Week 4: Information and statistical inference-1
Week 5: Information and statistical inference-2
Week 6: Properties of measures of Information-1
<input type="radio"/> Measures of information <input checked="" type="radio"/> Chain rules <input type="radio"/> Shape of measures of information <input checked="" type="radio"/> Data processing inequality <input type="radio"/> Unit 6 notes
<input type="radio"/> Quiz : Weekly Assignment 6 <input type="radio"/> Weekly Feedback forms <input type="radio"/> Solution 6
Week 7: Properties of measures of Information-2 and Information Theoretic lower bounds
Week 8: Information theoretic lower bounds and Data Compression-1
Week 9: Data Compression-1 (Unit 9) and Data Compression-2 (Unit 10)
Week 10: Data Compression-2 (Unit 10) and Data Compression-3 (Unit 11)
Week 11: Channel coding and capacity (Unit 12) and Shannon's channel coding theorem proof (Unit 13)
Week 12: Shannon's channel coding theorem proof (Unit 13) and Gaussian channels (Unit 14)
VIDEO DOWNLOAD
Live Session

Weekly Assignment 6

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-10-28, 23:59 IST.

Note: All the questions below may have multiple correct answers. There is no negative making for wrong choices made. However, partial marking is considered only when none of the marked choices is wrong.

Unless explicitly mentioned, assume that probability distributions are defined over a finite alphabet.

1) Let P and Q be two distributions over a finite alphabet \mathcal{X} . Which of the following are true? 1 point

- $\sum_{x \in \mathcal{X}} P(x) \log \frac{1}{Q(x)} \leq H(P)$
- $\sum_{x \in \mathcal{X}} P(x) \log \frac{1}{Q(x)} \geq H(P)$
- If P is a uniform distribution and $Q = (1, 0, \dots, 0)$, then $D(P||Q) = \log |\mathcal{X}|$.
- If P is a uniform distribution and $Q = (1, 0, \dots, 0)$, then $D(P||Q) > \log |\mathcal{X}|$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $\sum_{x \in \mathcal{X}} P(x) \log \frac{1}{Q(x)} \geq H(P)$
If P is a uniform distribution and $Q = (1, 0, \dots, 0)$, then $D(P||Q) > \log |\mathcal{X}|$.

2) Let X, Y be random variables with joint distribution $P_{X,Y}$. Which of the following are true? 1 point

- $I(X \wedge Y) = \mathbb{E}_{P_X}[D(P_{Y|X}||P_Y)]$
- $I(X \wedge Y) = \mathbb{E}_{P_Y}[D(P_{X|Y}||P_X)]$
- $I(X \wedge Y) = D(P_{X,Y}||P_X P_Y)$
- $I(Y \wedge X) = D(P_X P_Y||P_{X,Y})$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $I(X \wedge Y) = \mathbb{E}_{P_X}[D(P_{Y|X}||P_Y)]$
 $I(X \wedge Y) = \mathbb{E}_{P_Y}[D(P_{X|Y}||P_X)]$
 $I(X \wedge Y) = D(P_{X,Y}||P_X P_Y)$

3) Suppose X is a positive-valued random variable such that $\mathbb{E}[X] = 2$. Let $f(X) = X \log X$ and $g(X) = \log X$. Which of the following are true? 1 point

- $\mathbb{E}[f(X)] \leq 2$
- $\mathbb{E}[f(X)] \geq 2$
- $\mathbb{E}[f(X)] \geq \mathbb{E}[g(X)] + 1$
- $\mathbb{E}[f(X)] \leq \mathbb{E}[g(X)] + 1$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $\mathbb{E}[f(X)] \geq 2$
 $\mathbb{E}[f(X)] \geq \mathbb{E}[g(X)] + 1$

4) Let X be a random variable taking values in an alphabet \mathcal{X} . Let $Y = f(X)$, where $f: \mathcal{X} \rightarrow \mathcal{Y}$ is a deterministic function. Which of the following are true? 1 point

- $H(X) \leq H(X, Y)$
- $H(X) \geq H(X, Y)$
- $H(X) = H(Y)$
- $H(Y|X) = 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $H(X) \leq H(X, Y)$
 $H(X) \geq H(X, Y)$
 $H(Y|X) = 0$

5) Let P_1 and P_2 be probability distributions over alphabet \mathcal{X} such that $H(P_1) = H(P_2) = 2$. Let $P = \theta P_1 + (1 - \theta)P_2$, where $\theta \in [0, 1]$. Which of the following are true? 1 point

- $H(P) < 2$ for $\theta \in (0, 1/2)$
- P may not be a valid probability distribution, so $H(P)$ may not be well-defined.
- $H(P) \geq 2$ for every $\theta \in [0, 1]$
- $H(P) = 2$ for every $\theta \in [0, 1]$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $H(P) \geq 2$ for every $\theta \in [0, 1]$

6) Which of the following are true? 1 point

- $f(x) = x$ for $x > 0$ is a concave function.
- $f(x) = \log x$ for $x > 0$ is a concave function
- $f(x) = |x|$ for $x \in \mathbb{R}$ is a concave function
- $f(x) = \max\{x, x \log x\}$ for $x > 0$ is a concave function

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $f(x) = x$ for $x > 0$ is a concave function.
 $f(x) = \log x$ for $x > 0$ is a concave function

7) Let X, Y, Z be random variables with joint distribution $P_{X,Y,Z}$. Which of the following are true? 1 point

- $H(X, Y) = H(X) + H(X|Y)$
- $H(X, Y) \leq H(X) + H(Y)$
- $H(X, Y) \geq H(X|Z) + H(Y|X)$
- $H(X, Y) \leq H(X|Z) + H(Y|X)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $H(X, Y) \leq H(X) + H(Y)$
 $H(X, Y) \geq H(X|Z) + H(Y|X)$

8) Let X, Y, Z be random variables with joint probability distribution $P_{X,Y,Z}$. Which of the following are true? 1 point

- $I(X \wedge Y) \leq I(X \wedge Y, Z)$
- $I(X \wedge Y|Z) \geq I(X \wedge Y)$
- $I(X \wedge Y|Z) \leq I(X \wedge Y)$
- $I(X \wedge Y|Z) \leq I(X \wedge Y, Z)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $I(X \wedge Y) \leq I(X \wedge Y, Z)$
 $I(X \wedge Y|Z) \leq I(X \wedge Y, Z)$

9) Let X, Y be independent $\text{Ber}(1/2)$ random variables. Define $Z = X \oplus Y$, where \oplus is the XOR operator. Which of the following are true? 1 point

- $I(X \wedge Y) = 0$
- $I(X \wedge Z) = 0$
- $I(X \wedge Y|Z) = 0$
- $I(X \wedge Y|Z) = 1$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $I(X \wedge Y) = 0$
 $I(X \wedge Z) = 0$
 $I(X \wedge Y|Z) = 1$

10) Let X, Y, Z be random variables that form a Markov chain $X - Y - Z$. In the lectures, we saw the data processing inequality, $I(X \wedge Z) \leq I(X \wedge Y)$. 1 point
When does this inequality become a strict equality?

- If and only if $I(X \wedge Z) = I(X \wedge Y, Z)$
- If and only if $I(X \wedge Y|Z) = 0$
- If and only if X, Y, Z are mutually independent
- If and only if X, Y, Z form a Markov chain $X - Z - Y$

No, the answer is incorrect.
Score: 0

Accepted Answers:
If and only if $I(X \wedge Z) = I(X \wedge Y, Z)$
If and only if $I(X \wedge Y|Z) = 0$
If and only if X, Y, Z form a Markov chain $X - Z - Y$