

## Unit 6 - Week 5: Information and statistical inference-2

Course outline
How does an NPTEL online course work?
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Week 2: Uncertainty, compression, and entropy
Week 3: Randomness and entropy
Week 4: Information and statistical inference-1
Week 5: Information and statistical inference-2
<input type="radio"/> Information Theory Review 4 <input checked="" type="radio"/> Information per coin-loss <input type="radio"/> Multiple hypothesis testing <input type="radio"/> Error analysis of multiple hypothesis testing <input type="radio"/> Mutual information <input checked="" type="radio"/> Fano's inequality <input type="radio"/> Quiz : Weekly Assignment 5
<input type="radio"/> Weekly Feedback forms <input type="radio"/> Unit 5 Notes <input type="radio"/> Solution 5
Week 6: Properties of measures of Information-1
Week 7: Properties of measures of Information-2 and Information Theoretic lower bounds
Week 8: Information theoretic lower bounds and Data Compression-1
Week 9: Data Compression-1 (Unit 9) and Data Compression-2 (Unit 10)
Week 10: Data Compression-2 (Unit 10) and Data Compression-3 (Unit 11)
Week 11: Channel coding and capacity (Unit 12) and Shannon's channel coding theorem proof (Unit 13)
Week 12: Shannon's channel coding theorem proof (Unit 13) and Gaussian channels (Unit 14)
VIDEO DOWNLOAD
Live Session

## Weekly Assignment 5

The due date for submitting this assignment has passed.  
As per our records you have not submitted this assignment.

**Due on 2020-10-21, 23:59 IST.**

Note: All the questions below may have multiple correct answers. There is no negative marking for wrong choices. However, partial marking is considered only when none of the marked choices is wrong.

1) Let  $P = (\frac{1}{3}, \frac{1}{3})$  and  $Q = (\frac{1}{3} + \epsilon, \frac{1}{3} - \epsilon)$  be two distributions on alphabet  $\mathcal{X}$  of cardinality 2. Here,  $0 < \epsilon < 1/4$ . Which of the following are true? (You may use the inequality  $e^x \geq 1 + x$  for all  $x$ ) **1 point**

$$D(P||Q) = (\frac{1}{3} + \epsilon) \log \frac{\frac{1}{3} + \epsilon}{\frac{1}{3}} + (\frac{1}{3} - \epsilon) \log \frac{\frac{1}{3} - \epsilon}{\frac{1}{3}}$$

$$D(P||Q) = \frac{1}{2} \log \frac{1}{1-4\epsilon^2}$$

$$D(P||Q) \leq 2e^2$$

$$D(P||Q) \leq 4e^2$$

No, the answer is incorrect.  
Score: 0

Accepted Answers:

$$D(P||Q) = \frac{1}{2} \log \frac{1}{1-4\epsilon^2}$$

$$D(P||Q) \leq 4e^2$$

2) Let  $P = (1, 0)$  and  $Q = (1 - \epsilon, \epsilon)$  be two distributions on corresponding alphabet  $\mathcal{X} = \{1, 2\}$ . Suppose that  $0 < \epsilon < 1/4$ . Which of the following are true? **1 point**

$$D(P||Q) = \frac{1}{2} \log \frac{1}{1-4\epsilon^2}$$

$$D(P||Q) \leq 2e^2$$

$$D(P||Q) = \log \frac{1}{1-\epsilon}$$

$$D(P||Q) \leq 2\epsilon$$

No, the answer is incorrect.  
Score: 0

Accepted Answers:

$$D(P||Q) = \log \frac{1}{1-\epsilon}$$

$$D(P||Q) \leq 2\epsilon$$

3) Let  $P \equiv \mathcal{N}(\mu_1, I_d)$  and  $Q \equiv \mathcal{N}(\mu_{-1}, I_d)$ , where  $\mathcal{N}(\mu, C)$  denotes the  $d$ -dimensional Gaussian distribution with mean vector  $\mu$  and covariance matrix  $C$ . Also,  $\mu_1 = (1, 1, \dots, 1)$ ,  $\mu_{-1} = (-1, -1, \dots, -1)$ , and  $I_d$  is the  $d \times d$  identity matrix. Which of the following are true? **1 point**

$$D(P||Q) = 2$$

$$D(P||Q) \leq 2\sqrt{d}$$

$$D(P||Q) = d$$

$$D(P||Q) = 2d$$

No, the answer is incorrect.  
Score: 0

Accepted Answers:

$$D(P||Q) = 2d$$

4) Let  $\mathcal{X}, \mathcal{Y}$  be finite alphabets, and let  $f: \mathcal{X} \times \mathcal{Y} \rightarrow (0, \infty)$ ,  $g: \mathcal{X} \rightarrow (0, \infty)$ , and  $h: \mathcal{Y} \rightarrow (0, \infty)$  be positive-valued functions. Define  $x^* \triangleq \operatorname{argmax}_{x \in \mathcal{X}} f(x, y) h(y)$ . Which of the following are true? **1 point**

$$x^* = \operatorname{argmax}_{x \in \mathcal{X}} f(x, y)$$

$$x^* = \operatorname{argmax}_{x \in \mathcal{X}} e^{f(x, y)} h(y)$$

$$x^* = \operatorname{argmax}_{x \in \mathcal{X}} e^{-f(x, y)}$$

$$x^* = \operatorname{argmax}_{x \in \mathcal{X}} \frac{f(x, y)}{\sum_{x \in \mathcal{X}} g(x)}$$

No, the answer is incorrect.  
Score: 0

Accepted Answers:

$$x^* = \operatorname{argmax}_{x \in \mathcal{X}} f(x, y)$$

$$x^* = \operatorname{argmax}_{x \in \mathcal{X}} e^{f(x, y)} h(y)$$

$$x^* = \operatorname{argmax}_{x \in \mathcal{X}} \frac{f(x, y)}{\sum_{x \in \mathcal{X}} g(x)}$$

5) We are given a coin with an unknown probability of Heads. We toss the coin  $n$  times independently and estimate the probability of Heads using the principle of maximum likelihood. Let  $y = (y_1, \dots, y_n) \in \{0, 1\}^n$  be the result of the  $n$  tosses (with the mapping 'Heads'  $\rightarrow$  1, 'Tails'  $\rightarrow$  0). Then, maximum likelihood estimate of the probability of Heads is given by **1 point**

$$g_{ML}(y) = \operatorname{argmax}_{x \in [0, 1]} W_x^n(y)$$

where  $W_x^n$  denotes the  $n$ -fold product distribution of Bernoulli random variable with parameter  $x$ . Let  $n_y$  denote the number of times 1 appears in  $y = (y_1, \dots, y_n)$ . Which of the following are true?

(Recall that  $h(p) \triangleq -p \log p - (1-p) \log(1-p)$ .)

$$g_{ML}(y) = \operatorname{argmax}_{x \in [0, 1]} \binom{n}{n_y} x^{n_y} (1-x)^{n-n_y}$$

$$g_{ML}(y) = \operatorname{argmax}_{x \in [0, 1]} x^{n_y} (1-x)^{n-n_y}$$

$$g_{ML}(y) = \frac{n_y}{n}$$

$$g_{ML}(y) = h\left(\frac{n_y}{n}\right)$$

No, the answer is incorrect.  
Score: 0

Accepted Answers:

$$g_{ML}(y) = \operatorname{argmax}_{x \in [0, 1]} \binom{n}{n_y} x^{n_y} (1-x)^{n-n_y}$$

$$g_{ML}(y) = \operatorname{argmax}_{x \in [0, 1]} x^{n_y} (1-x)^{n-n_y}$$

$$g_{ML}(y) = \frac{n_y}{n}$$

6) Let  $\mathcal{X} = \{0, 1\}$  and assume that  $P_X(0) = 3/4$ ,  $P_X(1) = 1/4$ . Consider the binary hypothesis testing problem, where the hypotheses are **1 point**

$$\mathcal{H}_0: Y \sim W_0$$

$$\mathcal{H}_1: Y \sim W_1$$

Let  $P_x^*(P_X)$  be the minimum achievable (Bayesian) probability of error. Which of the following are true?

$$P_x^*(P_X) \text{ is achieved by the test } g(y) = \operatorname{argmax}_{x \in \mathcal{X}} W_x(y).$$

$$P_x^*(P_X) \text{ is achieved by the test } g(y) = \operatorname{argmax}_{x \in \mathcal{X}} P_X(x) W_x(y).$$

$$P_x^*(P_X) = \frac{1}{2} \left( 1 - \max_{A \subseteq \mathcal{Y}} (W_0(A) - W_1(A)) \right)$$

$$P_x^*(P_X) = \frac{3}{4} \left( 1 - \max_{A \subseteq \mathcal{Y}} (W_0(A) - \frac{1}{3} W_1(A)) \right)$$

No, the answer is incorrect.  
Score: 0

Accepted Answers:

$$P_x^*(P_X) \text{ is achieved by the test } g(y) = \operatorname{argmax}_{x \in \mathcal{X}} P_X(x) W_x(y).$$

$$P_x^*(P_X) = \frac{3}{4} \left( 1 - \max_{A \subseteq \mathcal{Y}} (W_0(A) - \frac{1}{3} W_1(A)) \right)$$

7) Let  $X$  and  $Y$  be independent  $\operatorname{Ber}(1/2)$  random variables defined on the alphabet  $\{0, 1\} \times \{0, 1\}$ . Which of the following are true? **1 point**

$$H(X|Y) = 0$$

$$H(X|Y) = 1$$

$$I(X \wedge Y) = 0$$

$$I(X \wedge Y) = 1$$

No, the answer is incorrect.  
Score: 0

Accepted Answers:

$$H(X|Y) = 1$$

$$I(X \wedge Y) = 0$$

8) Let  $X \in \{0, 1\}$  and  $Y \in \{0, 1\}$  be random variables with joint probability distribution  $P_{XY}$ . Moreover, suppose it is given that **1 point**

$X$  and  $Y$  are independent random variables.

$$I(X \wedge Y) = 1/2$$

$$H(Y) = \sum_{x, y} P_{XY}(x, y) \log \frac{1}{P_Y(y)}$$

$$H(X|Y) = 0$$

No, the answer is incorrect.  
Score: 0

Accepted Answers:

$$I(X \wedge Y) = 1/2$$

$$H(Y) = \sum_{x, y} P_{XY}(x, y) \log \frac{1}{P_Y(y)}$$

9) Let  $X$  and  $Y$  be  $\operatorname{Ber}(1/2)$  random variables defined on the alphabet  $\{0, 1\} \times \{0, 1\}$ , such that  $X = 1 - Y$ . Which of the following are true? **1 point**

$$H(X|Y) = 0$$

$$H(X|Y) = 1$$

$$I(X \wedge Y) = 0$$

$$I(X \wedge Y) = 1$$

No, the answer is incorrect.  
Score: 0

Accepted Answers:

$$H(X|Y) = 0$$

$$I(X \wedge Y) = 1$$

10) Let  $X \in \mathcal{X}$  and  $Y \in \mathcal{Y}$  be random variables defined on a finite alphabets with joint probability distribution  $P_{XY}$ . Moreover, suppose we know the following: **1 point**

- $X$  is distributed uniformly over  $\mathcal{X}$ .
- $\sum_{x \in \mathcal{X}} D(P_{Y|X=x} || P_Y) \leq 2 |\mathcal{X}|$ .
- There exists a function  $g: \mathcal{Y} \rightarrow \mathcal{X}$  such that  $P_{XY}(g(Y) \neq X) = 1/4$ .

What can we infer about,  $|\mathcal{X}|$ , the cardinality of  $\mathcal{X}$ ?

$$|\mathcal{X}| \leq 4$$

$$|\mathcal{X}| \leq 16$$

$$|\mathcal{X}| \geq 4$$

$$|\mathcal{X}| \geq 16$$

No, the answer is incorrect.  
Score: 0

Accepted Answers:

$$|\mathcal{X}| \leq 16$$