Unit 13 - Week 12: Shannon's channel coding theorem proof (Unit 13) and Gaussian channels (Unit 14)

Weekly Assignment 12

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-12-09, 23:59 IST.

Consider the following setup for Q1-Q3. $\mathcal{X}, \mathcal{Y}$ are finite alphabets. Let X be a random variable taking values in $\mathcal{X}$ with distribution P_X, and let $W : \mathcal{X} \to \mathcal{Y}$be a channel. For a given $x \in \mathcal{X}$, denote by W_x the probability distribution on $\mathcal{Y}$given by $W(\cdot|x)$. Let P_Y be the distribution of the channel output when the channel input is distributed as P_X, and let P_{XY} be the joint distribution of the channel input and the channel output. Suppose $A \subset \mathcal{X} \times \mathcal{Y}$ satisfies the following two properties: (i) $\sum_{(x,y)\in A} P_X(x)W(y|x) \geq 7/8$; (ii) for every $(x, y) \in A$, $\frac{W(y|x)}{P_Y(y)} \geq 32$.

1) Let $p_1 = \mathbb{E}_X\left[W_X(\{y : (X, y) \notin A\})\right]$. Which of the following is true? *1 point*

- $p_1 = P_X P_Y(A)$
- $p_1 = P_X P_Y(A^c)$
- $p_1 = P_{XY}(A)$
- $p_1 = P_{XY}(A^c)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$p_1 = P_{XY}(A^c)$

2) Let X, X' be i.i.d. copies from P_X. Let $p_2 = \mathbb{E}_{X,X'}\left[W_X(\{y : (X', y) \in A\})\right]$. Which of the following is true? *1 point*

- $p_1 = P_X P_Y(A)$
- $p_1 = P_X P_Y(A^c)$
- $p_1 = P_{XY}(A)$
- $p_1 = P_{XY}(A^c)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$p_1 = P_X P_Y(A)$

3) Let $\mathcal{M}$ be a message set. We would like to send a message selected uniformly at random from $\mathcal{M}$ by using the channel W once, such that the (average) probability of error is at most 1/4. Let M denote the maximum cardinality of $\mathcal{M}$ so that we can achieve this. Which of the following are true? *1 point*
($C(W)$ denotes the capacity of channel W.)

- $M \leq 2^{C(W)}$
- $M \geq 4$
- $M \leq 4$
- $M \geq 5$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$M \geq 4$
$M \geq 5$

4) Let $P_{X_1X_2X_3}$ be the joint distribution of random variables X_1, X_2, X_3 where $X_i \in \mathcal{X}$ for every $i \in \{1, 2, 3\}$. Let $W : \mathcal{X} \to \mathcal{Y}$ be a discrete memoryless channel. Suppose X_1, X_2 and X_3, when sent through the channel W, give outputs Y_1, Y_2 and Y_3, respectively. Which of the following are true? *1 point*

- Y_2 is independent of X_1.
- Y_2 is conditionally independent of X_1, given X_2.
- $H(Y_3|Y_1, Y_2, X_1, X_2, X_3) = H(Y_3|X_3)$
- $H(Y_3|Y_1, Y_2, X_1, X_2, X_3) = H(Y_3|X_2, X_3)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
Y_2 is conditionally independent of X_1, given X_2.
$H(Y_3|Y_1, Y_2, X_1, X_2, X_3) = H(Y_3|X_3)$
$H(Y_3|Y_1, Y_2, X_1, X_2, X_3) = H(Y_3|X_2, X_3)$

5) Let $\text{Geom}(p)$ denote the geometric distribution with parameter $p > 0$. Let $\text{Bin}(n, p)$ denote the binomial distribution with parameters $n \geq 1, p > 0$. Which of the following are true? *1 point*
(Recall that '$P \ll Q$' stands for 'P is absolutely continuous with respect to Q'.)

- $\text{Geom}(p) \ll \text{Bin}(n, p)$
- $\text{Bin}(n, p) \ll \text{Geom}(p)$
- $\text{Bin}(n, p) \ll \text{Bin}(2n, p)$
- $\text{Bin}(2n, p) \ll \text{Bin}(n, p)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\text{Bin}(n, p) \ll \text{Bin}(2n, p)$

6) Let P, Q be probability distributions on $\mathcal{X}$ such that $P \ll Q$. Let $g : \mathcal{X} \to \mathbb{R}$ be the Radon-Nikodym derivative of P with respect to Q. Which of the following are true? *1 point*

- $\mathbb{E}_P[X] = \mathbb{E}_Q[X]$
- $\mathbb{E}_P[X] = \mathbb{E}_Q[Xg(X)]$
- $\mathbb{E}_Q[X] = \mathbb{E}_P[Xg(X)]$
- $\mathbb{E}_P[g(X)] = \mathbb{E}_Q[g(X)]$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mathbb{E}_P[X] = \mathbb{E}_Q[Xg(X)]$

7) Let P_X be the probability distribution of a continuous random variable $X \in \mathbb{R}$. Suppose $\mathbb{E}[X^2] = 3$. What can we say about $h(X)$? *1 point*
($h(\cdot)$ denotes the differential entropy; $\log$ is base 2.)

- $h(X) \leq \frac{1}{2}\log 2\pi e$
- $h(X) = \frac{1}{2}\log 6\pi e$
- $h(X) \geq \frac{1}{2}\log 6\pi e$
- $h(X) \leq \log \sqrt{6\pi e}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$h(X) \leq \log \sqrt{6\pi e}$

8) Let $M(n, \epsilon)$ denote the maximum cardinality of the message set that can be sent by using a memoryless additive Gaussian noise channel ($\sigma^2 = 1$)n times, using codewords with power $P \leq 7$, such that the average probability of error is at most $\epsilon > 0$. What can we say about $M(n, \epsilon)$? *1 point*

- $M(n, \epsilon) \leq 2^{1.5n}$ for all n
- $M(n, \epsilon) \geq 2^{1.49n}$ for all n
- $M(n, \epsilon) \geq 2^{1.51n}$ for all sufficiently n
- $M(n, \epsilon) \geq 2^{1.49n}$ for all sufficiently n

No, the answer is incorrect.
Score: 0
Accepted Answers:
$M(n, \epsilon) \geq 2^{1.49n}$ *for all sufficiently* n

9) Suppose X is a real-valued random variable with distribution $P_X \equiv \mathcal{N}(0, 1)$. Let $\mathcal{E} = \{x \in \mathbb{R} : x^2 \leq 5\}$. Let $\bar{P}$ denote the probability distribution of X conditioned on the event $X \in \mathcal{E}$. That is, $\bar{P} = P_{X|X\in\mathcal{E}}$. Which of the following are true? *1 point*

- $P_X(\mathcal{E}) \leq 3/5$
- $P_X(\mathcal{E}) \geq 4/5$
- $\bar{P}(\mathcal{E}) \leq 0.9$
- $\bar{P}(A) = P_X(A)/P_X(\mathcal{E})$ for any event A.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$P_X(\mathcal{E}) \geq 4/5$

10) Suppose we have three parallel additive Gaussian noise channel channels with noise variances 1, 3 and 4. Suppose the overall power constraint for the three channels is $P = 1.5$. For attaining the capacity, how much power should be alloted to channel with variance 3? *1 point*

- 0
- 0.75
- 1
- 1.5

No, the answer is incorrect.
Score: 0
Accepted Answers:
0