

Unit 13 - Week 12: Shannon's channel coding theorem proof (Unit 13) and Gaussian channels (Unit 14)

Course outline

How does an NPTEL online course work?

Week 1- Information and probabilistic modeling

Week 2: Uncertainty, compression, and entropy

Week 3: Randomness and entropy

Week 4: Information and statistical inference-1

Week 5: Information and statistical inference-2

Week 6: Properties of measures of Information-1

Week 7: Properties of measures of Information-2 and Information Theoretic lower bounds

Week 8: Information theoretic lower bounds and Data Compression-1

Week 9: Data Compression-1 (Unit 9) and Data Compression-2 (Unit 10)

Week 10: Data Compression-2 (Unit 10) and Data Compression-3 (Unit 11)

Week 11: Channel coding and capacity (Unit 12) and Shannon's channel coding theorem proof (Unit 13)

Week 12: Shannon's channel coding theorem proof (Unit 13) and Gaussian channels (Unit 14)

● week 11 review

○ Converse proof for channel coding theorem

○ Additive Gaussian Noise channel

○ Mutual information and differential entropy

○ Channel coding theorem for Gaussian channel

● Parallel channels and water-filling

○ Quiz : Weekly Assignment 12

○ Unit-12 Notes

VIDEO DOWNLOAD

Live Session

Weekly Assignment 12

The due date for submitting this assignment has passed. **Due on 2020-12-09, 23:59 IST.**
As per our records you have not submitted this assignment.

Consider the following setup for Q1-Q3. \mathcal{X}, \mathcal{Y} are finite alphabets. Let X be a random variable taking values in \mathcal{X} with distribution P_X , and let $W : \mathcal{X} \rightarrow \mathcal{Y}$ be a channel. For a given $x \in \mathcal{X}$, denote by W_x the probability distribution on \mathcal{Y} given by $W(\cdot|x)$. Let P_Y be the distribution of the channel output when the channel input is distributed as P_X , and let P_{XY} be the joint distribution of the channel input and the channel output. Suppose $A \subset \mathcal{X} \times \mathcal{Y}$ satisfies the following two properties: (i) $\sum_{(x,y) \in A} P_X(x)W(y|x) \geq 7/8$; (ii) for every $(x, y) \in A$, $\frac{W(y|x)}{P_Y(y)} \geq 32$.

1) Let $p_1 = \mathbb{E}_X[W_X(\{y : (X, y) \notin A\})]$. Which of the following is true? **1 point**

- $p_1 = P_X P_Y(A)$
- $p_1 = P_X P_Y(A^c)$
- $p_1 = P_{XY}(A)$
- $p_1 = P_{XY}(A^c)$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$p_1 = P_{XY}(A^c)$

2) Let X, X' be i.i.d. copies from P_X . Let $p_2 = \mathbb{E}_{X, X'}[W_X(\{y : (X', y) \in A\})]$. Which of the following is true? **1 point**

- $p_1 = P_X P_Y(A)$
- $p_1 = P_X P_Y(A^c)$
- $p_1 = P_{XY}(A)$
- $p_1 = P_{XY}(A^c)$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$p_1 = P_X P_Y(A)$

3) Let \mathcal{M} be a message set. We would like to send a message selected uniformly at random from \mathcal{M} by using the channel W once, such that the (average) probability of error is at most 1/4. Let M denote the maximum cardinality of \mathcal{M} so that we can achieve this. Which of the following are true? ($C(W)$ denotes the capacity of channel W .) **1 point**

- $M \leq 2^{C(W)}$
- $M \geq 4$
- $M \leq 4$
- $M \geq 5$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$M \geq 4$

$M \geq 5$

4) Let P_{X_1, X_2, X_3} be the joint distribution of random variables X_1, X_2, X_3 where $X_i \in \mathcal{X}$ for every $i \in \{1, 2, 3\}$. Let $W : \mathcal{X} \rightarrow \mathcal{Y}$ be a discrete memoryless channel. Suppose X_1, X_2 and X_3 , when sent through the channel W , give outputs Y_1, Y_2 and Y_3 , respectively. Which of the following are true? **1 point**

- Y_2 is independent of X_1 .
- Y_2 is conditionally independent of X_1 , given X_2 .
- $H(Y_3|Y_1, Y_2, X_1, X_2, X_3) = H(Y_3|X_3)$
- $H(Y_3|Y_1, Y_2, X_1, X_2, X_3) = H(Y_3|X_2, X_3)$

No, the answer is incorrect.

Score: 0

Accepted Answers:

Y_2 is conditionally independent of X_1 , given X_2

$H(Y_3|Y_1, Y_2, X_1, X_2, X_3) = H(Y_3|X_3)$

$H(Y_3|Y_1, Y_2, X_1, X_2, X_3) = H(Y_3|X_2, X_3)$

5) Let $\text{Geom}(p)$ denote the geometric distribution with parameter $p > 0$. Let $\text{Bin}(n, p)$ denote the binomial distribution with parameters $n \geq 1, p > 0$. Which of the following are true? **1 point**

(Recall that ' $P \ll Q$ ' stands for ' P is absolutely continuous with respect to Q '.)

- $\text{Geom}(p) \ll \text{Bin}(n, p)$
- $\text{Bin}(n, p) \ll \text{Geom}(p)$
- $\text{Bin}(n, p) \ll \text{Bin}(2n, p)$
- $\text{Bin}(2n, p) \ll \text{Bin}(n, p)$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\text{Bin}(n, p) \ll \text{Bin}(2n, p)$

6) Let P, Q be probability distributions on \mathcal{X} such that $P \ll Q$. Let $g : \mathcal{X} \rightarrow \mathbb{R}$ be the Radon-Nikodym derivative of P with respect to Q . Which of the following are true? **1 point**

- $\mathbb{E}_P[X] = \mathbb{E}_Q[X]$
- $\mathbb{E}_P[X] = \mathbb{E}_Q[Xg(X)]$
- $\mathbb{E}_Q[X] = \mathbb{E}_P[Xg(X)]$
- $\mathbb{E}_P[g(X)] = \mathbb{E}_Q[g(X)]$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\mathbb{E}_P[X] = \mathbb{E}_Q[Xg(X)]$

7) Let P_X be the probability distribution of a continuous random variable $X \in \mathbb{R}$. Suppose $\mathbb{E}[X^2] = 3$. What can we say about $h(X)$? ($h(\cdot)$ denotes the differential entropy; \log is base 2.) **1 point**

- $h(X) \leq \frac{1}{2} \log 2\pi e$
- $h(X) = \frac{1}{2} \log 6\pi e$
- $h(X) \geq \frac{1}{2} \log 6\pi e$
- $h(X) \leq \log \sqrt{6\pi e}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$h(X) \leq \log \sqrt{6\pi e}$

8) Let $M(n, \epsilon)$ denote the maximum cardinality of the message set that can be sent by using a memoryless additive Gaussian noise channel ($\sigma^2 = 1$) n times, using codewords with power $P \leq 7$, such that the average probability of error is at most $\epsilon > 0$. What can we say about $M(n, \epsilon)$? **1 point**

- $M(n, \epsilon) \leq 2^{1.5n}$ for all n
- $M(n, \epsilon) \geq 2^{1.49n}$ for all n
- $M(n, \epsilon) \geq 2^{1.51n}$ for all sufficiently n
- $M(n, \epsilon) \geq 2^{1.49n}$ for all sufficiently n

No, the answer is incorrect.

Score: 0

Accepted Answers:

$M(n, \epsilon) \geq 2^{1.49n}$ for all sufficiently n

9) Suppose X is a real-valued random variable with distribution $P_X \equiv \mathcal{N}(0, 1)$. Let $\mathcal{E} = \{x \in \mathbb{R} : x^2 \leq 5\}$. Let \tilde{P} denote the probability distribution of X conditioned on the event $X \in \mathcal{E}$. That is, $\tilde{P} = P_{X|X \in \mathcal{E}}$. Which of the following are true? **1 point**

- $P_X(\mathcal{E}) \leq 3/5$
- $P_X(\mathcal{E}) \geq 4/5$
- $\tilde{P}(\mathcal{E}) \leq 0.9$
- $\tilde{P}(A) = P_X(A)/P_X(\mathcal{E})$ for any event A .

No, the answer is incorrect.

Score: 0

Accepted Answers:

$P_X(\mathcal{E}) \geq 4/5$

10) Suppose we have three parallel additive Gaussian noise channel channels with noise variances 1, 3 and 4. Suppose the overall power constraint for the three channels is $P = 1.5$. For attaining the capacity, how much power should be allotted to channel with variance 3? **1 point**

- 0
- 0.75
- 1
- 1.5

No, the answer is incorrect.

Score: 0

Accepted Answers:

0