

Unit 12 - Week 11: Channel coding and capacity (Unit 12) and Shannon's channel coding theorem proof (Unit 13)

Course outline

How does an NPTEL online course work?

Week 1: Information and probabilistic modeling

Week 2: Uncertainty, compression, and entropy

Week 3: Randomness and entropy

Week 4: Information and statistical inference-1

Week 5: Information and statistical inference-2

Week 6: Properties of measures of Information-1

Week 7: Properties of measures of Information-2 and Information Theoretic lower bounds

Week 8: Information theoretic lower bounds and Data Compression-1

Week 9: Data Compression-1 (Unit 9) and Data Compression-2 (Unit 10)

Week 10: Data Compression-2 (Unit 10) and Data Compression-3 (Unit 11)

Week 11: Channel coding and capacity (Unit 12) and Shannon's channel coding theorem proof (Unit 13)

Repetition code

Channel capacity

Sphere packing bound for BSC

Random coding bound for BSC

Random coding bound for general channel

Quiz : Weekly Assignment 11

Unit-11 notes

Solution 11

Week 12: Shannon's channel coding theorem proof (Unit 13) and Gaussian channels (Unit 14)

VIDEO DOWNLOAD

Live Session

Weekly Assignment 11

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-12-02, 23:59 IST.

- 1) Consider a channel $W : \{0, 1\} \rightarrow \{0, 1\}$ that works as follows: A bit is first passed through a BSC(0.1) channel and then passed through a BSC(0.5) channel. What is the capacity of channel W ? (BSC(δ) denotes a binary symmetric channel with probability of flip δ .) 1 point
- 0.05
 0.5
 0.1
 0
- No, the answer is incorrect.
Score: 0
Accepted Answers:
0
- 2) Let W be a BSC(δ) channel where $\delta > 1/2$. Suppose our message set is $\{0, 1\}$ and we encode a message using repetition code of length n (where n is odd). Let N denote the number of 1's observed at the output of W^n . Which of the following is the maximum likelihood decoder rule? 1 point
- Declare 1 if $N > n - N$, declare 0 otherwise
 Declare 0 if $N > n - N$, declare 1 otherwise
 Declare 1 if $N < n$, declare 0 otherwise
 Declare 0 if $N < n$, declare 1 otherwise
- No, the answer is incorrect.
Score: 0
Accepted Answers:
Declare 0 if $N > n - N$, declare 1 otherwise
- 3) Let Z_1, \dots, Z_n be i.i.d. Ber(δ) random variables. What can we say about $\mathbb{P}(\sum_{i=1}^n Z_i \geq n/2)$? 1 point
- $\mathbb{P}(\sum_{i=1}^n Z_i \geq n/2) = \mathbb{P}(|\frac{1}{n} \sum_{i=1}^n Z_i - \delta| \geq \frac{1}{2} - \delta)$
 $\mathbb{P}(\sum_{i=1}^n Z_i \geq n/2) \leq \mathbb{P}(|\frac{1}{n} \sum_{i=1}^n Z_i - \delta| \geq \frac{1}{2} - \delta)$
 $\mathbb{P}(\sum_{i=1}^n Z_i \geq n/2) \leq \frac{\delta(1-\delta)}{n(1/2-\delta)^2}$
 $\mathbb{P}(\sum_{i=1}^n Z_i \geq n/2) \leq 2\delta$
- No, the answer is incorrect.
Score: 0
Accepted Answers:
 $\mathbb{P}(\sum_{i=1}^n Z_i \geq n/2) \leq \mathbb{P}(|\frac{1}{n} \sum_{i=1}^n Z_i - \delta| \geq \frac{1}{2} - \delta)$
 $\mathbb{P}(\sum_{i=1}^n Z_i \geq n/2) \leq \frac{\delta(1-\delta)}{n(1/2-\delta)^2}$
 $\mathbb{P}(\sum_{i=1}^n Z_i \geq n/2) \leq 2\delta$
- 4) For a channel W , let $M(n, \epsilon)$ denote the maximum size of a code of length n such that the average probability of error is at most ϵ . Which of the following are true? (BSC denotes a binary symmetric channel, BEC denotes a binary erasure channel.) 1 point
- For BSC(1/2) channel, $M(1, 1/4) = 1$.
 For BSC(1/2) channel, $M(2, 1/4) \geq 2$.
 For BEC(1/2) channel, $M(1, 1/4) = 1$.
 For BEC(1/2) channel, $M(2, 1/4) \geq 2$.
- No, the answer is incorrect.
Score: 0
Accepted Answers:
For BSC(1/2) channel, $M(1, 1/4) = 1$.
For BEC(1/2) channel, $M(2, 1/4) \geq 2$.
- 5) For a channel W , a rate R is ϵ -achievable if $M(n, \epsilon) \geq 2^{nR}$ for all sufficiently large n . Denote by $C_\epsilon(W)$ the supremum of all rates R that are ϵ -achievable for a channel W . Which of the following are true? 1 point
- $C_{0.3}(W) \geq C_{0.4}(W)$
 $C_{0.3}(W) \leq C_{0.4}(W)$
 $C_{0.3}(W) = C_{0.4}(W)$
 $C_{0.3}(W) \geq 0$
- No, the answer is incorrect.
Score: 0
Accepted Answers:
 $C_{0.3}(W) \leq C_{0.4}(W)$
 $C_{0.3}(W) \geq 0$
- 6) Suppose X , which is uniformly distributed in $\{0, 1\}$, is given as an input to a BEC(δ) channel. What is the output distribution Q ? 1 point
- $Q = (1 - \delta, 2\delta, 1 - \delta)$
 $Q = (1 - \delta, 0, \delta)$
 $Q = ((1 - \delta)/2, \delta, (1 - \delta)/2)$
 $Q = (1 - \frac{\delta}{2}, \delta, 1 - \frac{\delta}{2})$
- No, the answer is incorrect.
Score: 0
Accepted Answers:
 $Q = ((1 - \delta)/2, \delta, (1 - \delta)/2)$
- 7) Consider a channel $W : \{1, 2, 3\} \rightarrow \{1, 2, 3\}$ such that $W_1 = (1 - \delta, \delta/2, \delta/2)$, $W_2 = (\delta/2, 1 - \delta, \delta/2)$, $W_3 = (\delta/2, \delta/2, 1 - \delta)$, where W_i denotes the output probability distribution when the input to the channel W is i . Which of the following is true? ($C(W)$ denotes the capacity of channel W , h denotes the binary entropy function.) 1 point
- $C(W) = \log 3 - h(\delta)$
 $C(W) = \log 3 + 1 - h(\delta)$
 $C(W) = \log 3 - \delta - h(2\delta)$
 $C(W) = \log 3 - \delta - h(\delta)$
- No, the answer is incorrect.
Score: 0
Accepted Answers:
 $C(W) = \log 3 - \delta - h(\delta)$
- 8) Let \mathcal{M} be the set of messages, where $|\mathcal{M}| = 64$. Suppose, for a channel W , there is an n -length code which can send these messages with average probability of error at most 0.1. Let \mathcal{M}' denote the set of messages for which the probability of error is at most 0.3. What can be inferred about $|\mathcal{M}'|$? 1 point
- $|\mathcal{M}'| \leq 22$
 $|\mathcal{M}'| \geq 23$
 $|\mathcal{M}'| \leq 42$
 $|\mathcal{M}'| \geq 43$
- No, the answer is incorrect.
Score: 0
Accepted Answers:
 $|\mathcal{M}'| \geq 23$
 $|\mathcal{M}'| \geq 43$
- 9) Let $C_k = \sum_{i=0}^k \binom{n}{i}$, where $k \leq n/2$. Which of the following are true? (h denotes the binary entropy function.) 1 point
- $C_k \leq 2^{nh(k/n)}$
 $C_k \leq (k+1) \binom{n}{k}$
 $C_k \leq \frac{n!}{k!(n-k)!}$
 $C_k \leq \frac{n^n}{k^n(n-k)^n}$
- No, the answer is incorrect.
Score: 0
Accepted Answers:
 $C_k \leq 2^{nh(k/n)}$
 $C_k \leq (k+1) \binom{n}{k}$
 $C_k \leq \frac{n^n}{k^n(n-k)^n}$
- 10) Let $W : \mathcal{X} \rightarrow \mathcal{Y}$ be a channel. Let P_1 and P_2 be probability distributions on \mathcal{X} , such that $I(P_1; W) \geq 2$, $I(P_2; W) \geq 3$. Which of the following are true? ($C(W)$ denotes the capacity of channel W .) 1 point
- $I(\frac{1}{3}P_1 + \frac{2}{3}P_2; W) \leq 2.6$
 $I(\frac{1}{2}P_1 + \frac{1}{2}P_2; W) \geq 2.5$
 $C(W) \geq 3$
 $C(W) \leq 4$
- No, the answer is incorrect.
Score: 0
Accepted Answers:
 $I(\frac{1}{2}P_1 + \frac{1}{2}P_2; W) \geq 2.5$
 $C(W) \geq 3$