Unit 12 - Week 11: Channel coding and capacity (Unit 12) and Shannon's channel coding theorem proof (Unit 13)

Weekly Assignment 11

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-12-02, 23:59 IST.

1) Consider a channel $W:\{0,1\}\to\{0,1\}$ that works as follows: A bit is first passed through a $\mathrm{BSC}(0.1)$ channel and then passed through a $\mathrm{BSC}(0.5)$ channel. What is the capacity of channel W? *1 point*
($\mathrm{BSC}(\delta)$ denotes a binary symmetric channel with probability of flip δ.)

- 0.05
- 0.5
- 0.1
- 0

No, the answer is incorrect.
Score: 0
Accepted Answers:
0

2) Let W be a BSC(δ) channel where $\delta > 1/2$. Suppose our message set is $\{0,1\}$ and we encode a message using repetition code of length n (where n is odd). Let N denote the number of 1's observed at the output of W^n. Which of the following is the maximum likelihood decoder rule? *1 point*

- Declare 1 if $N > n - N$, declare 0 otherwise
- Declare 0 if $N > n - N$, declare 1 otherwise
- Declare 1 if $N < n$, declare 0 otherwise
- Declare 0 if $N < n$, declare 1 otherwise

No, the answer is incorrect.
Score: 0
Accepted Answers:
Declare 0 if $N > n - N$, declare 1 otherwise

3) Let $Z_1,\dots,Z_n$ be i.i.d. $\mathrm{Ber}(\delta)$ random variables. What can we say about $\mathbb{P}\left(\sum_{i=1}^n Z_i \ge n/2\right)$? *1 point*

- $\mathbb{P}\left(\sum_{i=1}^n Z_i \ge n/2\right) = \mathbb{P}\left(\left|\frac{1}{n}\sum_{i=1}^n Z_i - \delta\right| \ge \frac{1}{2} - \delta\right)$
- $\mathbb{P}\left(\sum_{i=1}^n Z_i \ge n/2\right) \le \mathbb{P}\left(\left|\frac{1}{n}\sum_{i=1}^n Z_i - \delta\right| \ge \frac{1}{2} - \delta\right)$
- $\mathbb{P}\left(\sum_{i=1}^n Z_i \ge n/2\right) \le \frac{\delta(1-\delta)}{n(1/2-\delta)^2}$
- $\mathbb{P}\left(\sum_{i=1}^n Z_i \ge n/2\right) \le 2\delta$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mathbb{P}\left(\sum_{i=1}^n Z_i \ge n/2\right) \le \mathbb{P}\left(\left|\frac{1}{n}\sum_{i=1}^n Z_i - \delta\right| \ge \frac{1}{2} - \delta\right)$
$\mathbb{P}\left(\sum_{i=1}^n Z_i \ge n/2\right) \le \frac{\delta(1-\delta)}{n(1/2-\delta)^2}$
$\mathbb{P}\left(\sum_{i=1}^n Z_i \ge n/2\right) \le 2\delta$

4) For a channel W, let $M(n,\epsilon)$ denote the maximum size of a code of length n such that the average probability of error is at most ϵ. Which of the following are true? *1 point*
(BSC denotes a binary symmetric channel, BEC denotes a binary erasure channel.)

- For BSC(1/2) channel, $M(1,1/4) = 1$.
- For BSC(1/2) channel, $M(2,1/4) \ge 2$.
- For BEC(1/2) channel, $M(1,1/4) = 1$.
- For BEC(1/2) channel, $M(2,1/4) \ge 2$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
For BSC(1/2) channel, $M(1,1/4) = 1$.
For BEC(1/2) channel, $M(2,1/4) \ge 2$.

5) For a channel W, a rate R is ϵ-achievable if $M(n,\epsilon) \ge 2^{nR}$ for all sufficiently large n. Denote by $C_\epsilon(W)$ the supremum of all rates R that are ϵ-achievable for a channel W. Which of the following are true? *1 point*

- $C_{0.3}(W) \ge C_{0.4}(W)$
- $C_{0.3}(W) \le C_{0.4}(W)$
- $C_{0.3}(W) = C_{0.4}(W)$
- $C_{0.3}(W) \ge 0$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$C_{0.3}(W) \le C_{0.4}(W)$
$C_{0.3}(W) \ge 0$

6) Suppose X, which is uniformly distributed in $\{0,1\}$, is given as an input to a BEC(δ) channel. What is the output distribution Q? *1 point*

- $Q = (1-\delta, 2\delta, 1-\delta)$
- $Q = (1-\delta, 0, \delta)$
- $Q = ((1-\delta)/2, \delta, (1-\delta)/2)$
- $Q = (1-\frac{\delta}{2}, \delta, 1-\frac{\delta}{2})$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$Q = ((1-\delta)/2, \delta, (1-\delta)/2)$

7) Consider a channel $W:\{1,2,3\}\to\{1,2,3\}$ such that: $W_1 = (1-\delta, \delta/2, \delta/2), W_2 = (\delta/2, 1-\delta, \delta/2), W_3 = (\delta/2, \delta/2, 1-\delta)$, where W_i denotes the output probability distribution when the input to the channel W is i. Which of the following is true? *1 point*
($C(W)$ denotes the capacity of channel W, h denotes the binary entropy function.)

- $C(W) = \log 3 - h(\delta)$
- $C(W) = \log 3 + 1 - h(\delta)$
- $C(W) = \log 3 - \delta - h(2\delta)$
- $C(W) = \log 3 - \delta - h(\delta)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$C(W) = \log 3 - \delta - h(\delta)$

8) Let $\mathcal{M}$ be the set of messages, where $|\mathcal{M}| = 64$. Suppose, for a channel W, there is an n-length code which can send these messages with average probability of error at most 0.1. Let $\mathcal{M}'$ denote the set of messages for which the probability of error is at most 0.3. What can be inferred about $|\mathcal{M}'|$? *1 point*

- $|\mathcal{M}'| \le 22$
- $|\mathcal{M}'| \ge 23$
- $|\mathcal{M}'| \le 42$
- $|\mathcal{M}'| \ge 43$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$|\mathcal{M}'| \ge 23$
$|\mathcal{M}'| \ge 43$

9) Let $C_k = \sum_{i=0}^k \binom{n}{i}$, where $k \le n/2$. Which of the following are true?(h denotes the binary entropy function.) *1 point*

- $C_k \le 2^{nh(k/n)}$
- $C_k \le (k+1)\binom{n}{k}$
- $C_k \le \frac{n!}{k!(n-k)!}$
- $C_k \le \frac{n^n}{k^k(n-k)^k}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$C_k \le 2^{nh(k/n)}$
$C_k \le (k+1)\binom{n}{k}$
$C_k \le \frac{n^n}{k^k(n-k)^k}$

10) Let $W:\mathcal{X}\to\mathcal{Y}$ be a channel. Let P_1 and P_2 be probability distributions on $\mathcal{X}$, such that $I(P_1;W) \ge 2, I(P_2;W) \ge 3$. Which of the following are true? *1 point*
($C(W)$ denotes the capacity of channel W.)

- $I(\frac{1}{3}P_1 + \frac{2}{3}P_2; W) \le 2.6$
- $I(\frac{1}{2}P_1 + \frac{1}{2}P_2; W) \ge 2.5$
- $C(W) \ge 3$
- $C(W) \le 4$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$I(\frac{1}{2}P_1 + \frac{1}{2}P_2; W) \ge 2.5$
$C(W) \ge 3$