

Unit 2 - Week 1- Information and probabilistic modeling

Course outline
How does an NPTEL online course work?
Week 1- Information and probabilistic modeling <ul style="list-style-type: none"> <input checked="" type="radio"/> What is information? <input type="radio"/> How to model uncertainty? <input type="radio"/> Basic concepts of probability <input checked="" type="radio"/> Estimates of random variables <input type="radio"/> Limit theorems <input type="radio"/> Quiz : Weekly Assignment 1 <input type="radio"/> Unit 1 - Notes <input checked="" type="radio"/> Solution 1
Week 2: Uncertainty, compression, and entropy
Week 3: Randomness and entropy
Week 4: Information and statistical inference-1
Week 5: Information and statistical inference-2
Week 6: Properties of measures of Information-1
Week 7: Properties of measures of Information-2 and Information Theoretic lower bounds
Week 8: Information theoretic lower bounds and Data Compression-1
Week 9: Data Compression-1 (Unit 9) and Data Compression-2 (Unit 10)
Week 10: Data Compression-2 (Unit 10) and Data Compression-3 (Unit 11)
Week 11: Channel coding and capacity (Unit 12) and Shannon's channel coding theorem proof (Unit 13)
Week 12: Shannon's channel coding theorem proof (Unit 13) and Gaussian channels (Unit 14)
VIDEO DOWNLOAD
Live Session

Weekly Assignment 1

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

Due on 2020-09-30, 23:59 IST.

1) A box contains one coupon labelled 1, two coupons labelled 2, and so on up to 10 coupons labelled 10. Two distinct coupons are drawn uniformly at random from the box. The cardinality of sample space (set of all possible outcomes) \mathcal{X} is **1 point**

- 55
 90
 54
 100

No, the answer is incorrect.
Score: 0

Accepted Answers:
54

2) Suppose A_1, A_2, \dots, A_n are events with non-zero probabilities. Which of the following is/are always true? (Choose all that apply) **1 point**

- A_1, A_2, \dots, A_n are mutually independent if $\mathbb{P}(A_i \cap A_j) = \mathbb{P}(A_i) \cdot \mathbb{P}(A_j)$, for every $i, j \in \{1, 2, \dots, n\}$
 A_1, A_2, \dots, A_n are mutually independent if $\mathbb{P}(\bigcap_{i \in J} A_i) = \prod_{i \in J} \mathbb{P}(A_i)$ for every subset $J \subseteq \{1, 2, \dots, n\}$
 $\mathbb{P}(\bigcap_{i=1}^n A_i) = \prod_{i=1}^n \mathbb{P}(A_i | \bigcap_{j=1}^{i-1} A_j)$

A_1, A_2 are mutually independent if they are mutually exclusive (i.e. $A_1 \cap A_2$ is empty)

No, the answer is incorrect.
Score: 0

Accepted Answers:

A_1, A_2, \dots, A_n are mutually independent if $\mathbb{P}(\bigcap_{i \in J} A_i) = \prod_{i \in J} \mathbb{P}(A_i)$ for every subset $J \subseteq \{1, 2, \dots, n\}$

$\mathbb{P}(\bigcap_{i=1}^n A_i) = \prod_{i=1}^n \mathbb{P}(A_i | \bigcap_{j=1}^{i-1} A_j)$

Questions 3-6 are based on the following scenario:

A coin with probability of Heads 0.9 is tossed 100 times. This will generate a sequence of Heads and Tails of length 100.

3) Let \mathcal{E}_{90} be the set of sequences with 90 Heads and 10 Tails. How many sequences does \mathcal{E}_{90} contain? **1 point**

- 90!
 $\binom{100}{90}$
 $\binom{100}{90} 90!$
 $(0.9)^{90}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $\binom{100}{90}$

4) Let \mathcal{E}_{100} be the set of sequences with 100 Heads. How many sequences does \mathcal{E}_{100} contain? **1 point**

- 100!
 100^{100}
 1
 $(0.9)^{100}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
1

5) Between events \mathcal{E}_{90} and \mathcal{E}_{100} , which event does occur with higher probability? **1 point**

- \mathcal{E}_{90}
 \mathcal{E}_{100}
 Both events have equal probability of occurrence

No, the answer is incorrect.
Score: 0

Accepted Answers:
 \mathcal{E}_{90}

6) Let \mathbf{a} be a sequence with 90 Heads and 10 Tails (i.e. $\mathbf{a} \in \mathcal{E}_{90}$). Let \mathbf{b} be a sequence with 100 heads (i.e. $\mathbf{b} \in \mathcal{E}_{100}$). Between \mathbf{a} and \mathbf{b} , which sequence does occur with higher probability? **1 point**

- \mathbf{a}
 \mathbf{b}
 Both sequences have equal probability of occurrence

No, the answer is incorrect.
Score: 0

Accepted Answers:
 \mathbf{b}

7) Suppose a coin with probability of Heads p is tossed repeatedly until the first Heads appears. What is the expected number of tosses required? **1 point**

- p
 $1-p$
 $\frac{1}{p}$
 $\frac{1}{p(1-p)}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $\frac{1}{p}$

8) Let X_1, \dots, X_n be independent and identically distributed random variables with distribution **Exponential(1)** and $Y_n = \sum_{i=1}^n X_i$. Let n^* be the least n required to ensure that $\Pr\{Y_n \geq 10^6\} \geq 3/4$. Using Markov's inequality, a lower bound on n^* is ($\lceil x \rceil$ denotes the smallest integer greater than or equal to x) **1 point**

- $\frac{3}{4} \times 10^6$
 1×10^6
 $\lceil \frac{4}{3} \times 10^6 \rceil$
 $\frac{1}{4} \times 10^7$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $\frac{3}{4} \times 10^6$

9) By Chebyshev's inequality, the number of Heads that will be seen when an unbiased coin is tossed 10^8 times lies in the interval _____ with probability at least $1 - \delta$, where $0 < \delta < 1$. **1 point**

- $\left[\frac{10^8}{2} - \frac{10^4}{2\sqrt{\delta}}, \frac{10^8}{2} + \frac{10^4}{2\sqrt{\delta}} \right]$
 $\left[\frac{10^8}{2} - \frac{10^4}{2\delta}, \frac{10^8}{2} + \frac{10^4}{2\delta} \right]$
 $\left[\frac{10^8}{2} - \frac{10^4}{4\sqrt{\delta}}, \frac{10^8}{2} + \frac{10^4}{4\sqrt{\delta}} \right]$
 $\left[\frac{10^8}{2} - \frac{10^4}{8\sqrt{\delta}}, \frac{10^8}{2} + \frac{10^4}{8\sqrt{\delta}} \right]$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$\left[\frac{10^8}{2} - \frac{10^4}{2\sqrt{\delta}}, \frac{10^8}{2} + \frac{10^4}{2\sqrt{\delta}} \right]$
 $\left[\frac{10^8}{2} - \frac{10^4}{2\delta}, \frac{10^8}{2} + \frac{10^4}{2\delta} \right]$

10) A coin with probability of Heads 0.3 is tossed 1000 times independently. By central limit theorem, the probability that we see at least 800 Heads is roughly $Q(t)$. The value of t is

Q denotes the Gaussian-tail probability, i.e. $Q(u) = \int_u^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$

- $\frac{500}{21\sqrt{10}}$
 $\frac{300}{\sqrt{210}}$
 $\frac{300}{21\sqrt{10}}$
 $\frac{500}{\sqrt{210}}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
 $\frac{500}{\sqrt{210}}$