Unit 2 - Week 1- Information and probabilistic modeling

Weekly Assignment 1

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-09-30, 23:59 IST.

1) A box contains one coupon labelled 1, two coupons labelled 2, and so on up to 10 coupons labelled 10. Two distinct coupons are drawn uniformly at random from the box. The cardinality of sample space (set of all possible outcomes) $\mathcal{X}$ is *1 point*

- 55
- 90
- 54
- 100

No, the answer is incorrect.
Score: 0
Accepted Answers:
54

2) Suppose $A_1, A_2, \ldots, A_n$ are events with non-zero probabilities. Which of the following is/are always true? (Choose all that apply) *1 point*

- $A_1, A_2, \ldots, A_n$ are mutually independent if $\mathbb{P}(A_i \cap A_j) = \mathbb{P}(A_i) \cdot \mathbb{P}(A_j)$, for every $i, j \in \{1, 2, \ldots, n\}$
- $A_1, A_2, \ldots, A_n$ are mutually independent if $\mathbb{P}(\bigcap_{i \in J} A_i) = \prod_{i \in J} \mathbb{P}(A_i)$ for every subset $J \subseteq \{1, 2, \ldots, n\}$
- $\mathbb{P}(\bigcap_{i=1}^{n} A_i) = \prod_{i=1}^{n} \mathbb{P}(A_i \mid \bigcap_{j=1}^{i-1} A_j)$
- A_1, A_2 are mutually independent if they are mutually exclusive (i.e. $A_1 \cap A_2$ is empty)

No, the answer is incorrect.
Score: 0
Accepted Answers:
$A_1, A_2, \ldots, A_n$ *are mutually independent if*
$\mathbb{P}(\bigcap_{i \in J} A_i) = \prod_{i \in J} \mathbb{P}(A_i)$ for every subset $J \subseteq \{1, 2, \ldots, n\}$
$\mathbb{P}(\bigcap_{i=1}^{n} A_i) = \prod_{i=1}^{n} \mathbb{P}(A_i \mid \bigcap_{j=1}^{i-1} A_j)$

Questions 3-6 are based on the following scenario:
A coin with probability of Heads 0.9 is tossed 100 times. This will generate a sequence of Heads and Tails of length 100.

3) Let $\mathcal{E}_{90}$ be the set of sequences with 90 Heads and 10 Tails. How many sequences does $\mathcal{E}_{90}$ contain? *1 point*

- $90!$
- $\binom{100}{90}$
- $\binom{100}{90} 90!$
- $(0.9)^{90}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\binom{100}{90}$

4) Let $\mathcal{E}_{100}$ be the set of sequences with 100 Heads. How many sequences does $\mathcal{E}_{100}$ contain? *1 point*

- $100!$
- 100^{100}
- 1
- $(0.9)^{100}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
1

5) Between events $\mathcal{E}_{90}$ and $\mathcal{E}_{100}$, which event does occur with higher probability? *1 point*

- $\mathcal{E}_{90}$
- $\mathcal{E}_{100}$
- Both events have equal probability of occurence

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mathcal{E}_{90}$

6) Let $\mathbf{a}$ be a sequence with 90 Heads and 10 Tails (i.e. $\mathbf{a} \in \mathcal{E}_{90}$). Let $\mathbf{b}$ be a sequence with 100 heads (i.e. $\mathbf{b} \in \mathcal{E}_{100}$). Between $\mathbf{a}$ and $\mathbf{b}$, which sequence does occur with higher probability? *1 point*

- $\mathbf{a}$
- $\mathbf{b}$
- Both sequences have equal probability of occurence

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mathbf{b}$

7) Suppose a coin with probability of Heads p is tossed repeatedly until the first Heads appears. What is the expected number of tosses required? *1 point*

- p
- $1 - p$
- $\frac{1}{p}$
- $\frac{1}{p(1-p)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{1}{p}$

8) Let $X_1, \ldots, X_n$ be independent and identically distributed random variables with distribution $\texttt{Exponential}(1)$ and $Y_n = \sum_{i=1}^{n} X_i$. Let n^* be the least n required to ensure that $\Pr\{Y_n \geq 10^6\} \geq 3/4$. Using Markov's inequality, a lower bound on n^* is *1 point*
($\lceil x \rceil$ denotes the smallest integer greater than or equal to x)

- $\frac{3}{4} \times 10^6$
- 1×10^6
- $\lceil \frac{4}{3} \times 10^6 \rceil$
- $\frac{1}{4} \times 10^7$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{3}{4} \times 10^6$

9) By Chebyshev's inequality, the number of Heads that will be seen when an unbiased coin is tossed 10^8 times lies in the interval ______ with probability at least $1 - \delta$, where $0 < \delta < 1$. *1 point*

- $\left[\frac{10^8}{2} - \frac{10^4}{2\sqrt{\delta}}, \frac{10^8}{2} + \frac{10^4}{2\sqrt{\delta}}\right]$
- $\left[\frac{10^8}{2} - \frac{10^4}{2\delta}, \frac{10^8}{2} + \frac{10^4}{2\delta}\right]$
- $\left[\frac{10^8}{2} - \frac{10^4}{4\sqrt{\delta}}, \frac{10^8}{2} + \frac{10^4}{4\sqrt{\delta}}\right]$
- $\left[\frac{10^8}{2} - \frac{10^4}{8\sqrt{\delta}}, \frac{10^8}{2} + \frac{10^4}{8\sqrt{\delta}}\right]$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\left[\frac{10^8}{2} - \frac{10^4}{2\sqrt{\delta}}, \frac{10^8}{2} + \frac{10^4}{2\sqrt{\delta}}\right]$
$\left[\frac{10^8}{2} - \frac{10^4}{2\delta}, \frac{10^8}{2} + \frac{10^4}{2\delta}\right]$

10) A coin with probability of Heads 0.3 is tossed 1000 times independently. By central limit theorem, the probability that we see at least 800 Heads is roughly $Q(t)$. The value of t is *1 point*
(Q denotes the Gaussian-tail probablity, i.e. $Q(u) = \int_u^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$)

- $\frac{500}{21\sqrt{10}}$
- $\frac{300}{\sqrt{210}}$
- $\frac{300}{21\sqrt{10}}$
- $\frac{500}{\sqrt{210}}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{500}{\sqrt{210}}$