Unit 12 - Week 9 - Reproducing Kernel Hilbert Space and Regularization

Assignment 09

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-10-02, 23:59 IST.

Instructions:

1. **Attempt all questions.**
2. **Submission deadline**: 2nd October 2019 23:59 IST
3. **Solutions to be posted**: 3rd October 2019
4. Older browsers might show **unnecessary vertical bars** at the end of math equations.

1) Let V (assume completeness of V) consist of band-limited functions $f:\mathbb{R}\to\mathbb{R}$ expressible as $f(t)=\frac{1}{2\pi}\int_{-a}^{a}F(w)\exp(jwt)dw,$ where $F(w)$ is square-integrable. Endow V with the inner product $\langle f,g\rangle=\int_{\infty}^{\infty}f(t)g(t)dt$. Then V is a reproducible kernel Hilbert space (RKHS) with which of the following kernel? *3 points*

- $K(t,\tau)=\frac{\sin(a(t+\tau))}{\pi(t-\tau)}$
- $K(t,\tau)=\frac{\sin(a(t-\tau))}{\pi(t+\tau)}$
- $K(t,\tau)=\frac{\sin(a(t+\tau))}{\pi(t+\tau)}$
- $K(t,\tau)=\frac{\sin(a(t-\tau))}{\pi(t-\tau)}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$K(t,\tau)=\frac{\sin(a(t-\tau))}{\pi(t-\tau)}$

2) A reproducing kernel, if it exists, is unique. *2 points*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
True

3) L_2 is defined as set of all f such that $\int_{-\infty}^{\infty}\|f(x)\|^2dx<\infty$. Which of the following statements is true? *2 points*

- L_2 space is neither a Hilbert space nor an RKHS.
- L_2 space is a Hilbert space but not an RKHS.
- L_2 space is not a Hilbert space but an RKHS.
- L_2 space is a Hilbert space and an RKHS.

No, the answer is incorrect.
Score: 0
Accepted Answers:
L_2 space is a Hilbert space but not an RKHS.

4) Let $K(i,j)=\delta_{ij}$ where δ_{ij} is defined as follows: *3 points*

$$\delta_{ij}=\begin{cases}1 & i=j\\ 0 & i\neq j.\end{cases}$$

Then, K is the reproducible kernel of $\mathcal{H}$ as K satisfies

- $\forall j\in\mathbb{N}, K(.,j)=(0,0,\dots,1,0,0)\in\mathcal{H}$ (1 at the j-th place) and $\forall j\in\mathbb{N},\forall x=(x_i)_{i\in\mathbb{N}}\in\mathcal{H},\langle x,K(.,j)\rangle_{\mathcal{H}}=\sum_{i\in\mathbb{N}}x_i\delta^{\star}_{ij}=x_j$.
- $\forall j\in\mathbb{N}, K(.,j)=(0,0,\dots,1,0,0)\notin\mathcal{H}$ (1 at the j-th place) and $\forall j\in\mathbb{N},\forall x=(x_i)_{i\in\mathbb{N}}\in\mathcal{H},\langle x,K(.,j)\rangle_{\mathcal{H}}=\sum_{i\in\mathbb{N}}x_i\delta^{\star}_{ij}=x_j$.
- $\forall j\in\mathbb{N}, K(.,j)=(0,0,\dots,1,0,0)\notin\mathcal{H}$ (1 at the j-th place) and $\forall j\in\mathbb{N},\forall x=(x_i)_{i\in\mathbb{N}}\in\mathcal{H},\langle x,K(.,j)\rangle_{\mathcal{H}}=\sum_{i\in\mathbb{N}}x_i\delta^{\star}_{ij}\neq x_j$.
- $\forall j\in\mathbb{N}, K(.,j)=(0,0,\dots,1,0,0)\in\mathcal{H}$ (1 at the j-th place) and $\forall j\in\mathbb{N},\forall x=(x_i)_{i\in\mathbb{N}}\in\mathcal{H},\langle x,K(.,j)\rangle_{\mathcal{H}}=\sum_{i\in\mathbb{N}}x_i\delta^{\star}_{ij}\neq x_j$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\forall j\in\mathbb{N}, K(.,j)=(0,0,\dots,1,0,0)\in\mathcal{H}$ (1 at the j-th place) and $\forall j\in\mathbb{N},\forall x=(x_i)_{i\in\mathbb{N}}\in\mathcal{H},\langle x,K(.,j)\rangle_{\mathcal{H}}=\sum_{i\in\mathbb{N}}x_i\delta^{\star}_{ij}=x_j$.

5) (True/False): Classifying data points linearly using perceptron is a well-posed problem in Hadamard sense. *2 points*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
False

6) Let $(e_1,e_2,\dots,e_n)$ be an orthonormal basis in $\mathcal{H}$. Define $K(x,y)=\sum_{i=1}^{n}e_i(x)\bar{e}_i(y)$ ($\bar{e}_i$ is the complex conjugate of e_i). Then for any y in E (E is a non-empty abstract space), $K(.,y)=\sum_{i=1}^{n}\bar{e}_i(y)e_i(.)$ belongs to $\mathcal{H}$. For any function $\phi(.)=\sum_{i=1}^{n}\lambda_ie_i(.)$ in $\mathcal{H}$, compute $\langle\phi,K(.,y)\rangle_{\mathcal{H}}$ *2 points*

- $\phi(y)$
- $K(y)$
- $e_i(y)$
- $K(x,y)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\phi(y)$

7) In continuation with question 6, any finite dimensional Hilbert space of functions has a reproducing kernel. *2 points*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
True

8) Let $\mathcal{H}$ be a RKHS and $k(x,.)$ be a Mercer kernel. Then any function $f(.)\in\mathcal{H}$ can be decomposed into sum of two components, say $f_{||}$ and $f_{\perp}$. $f_{||}$ is in the span of the kernel functions $k(x_1,.),k(x_2,.)\dots k(x_l,.)$ and $f_{\perp}$ is orthogonal to the span of the kernel functions. Compute $f(x_j)$ *2 points*

- $f(x_j)=\sum_{i=1}^{l}a_ik(x_i,.)+\sum_{j=1}^{l}f_{\perp}(x_j)$
- $f(x_j)=\sum_j\sum_{i=1}^{l}a_ik(x_j,x_j)+f_{\perp}(x_j)$
- $f(x_j)=\sum_{j=1}^{l}\sum_{i=1}^{l}a_ik(x_j,x_j)$
- $f(x_j)=\sum_{i=1}^{l}a_ik(x_i,x_j)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$f(x_j)=\sum_{i=1}^{l}a_ik(x_i,x_j)$

9) Let $\mathcal{H}$ be a Hilbert space. Given any positive semi-definite kernel k, then we can construct a RKHS using $k(x,.)$. *2 points*

- True
- False

No, the answer is incorrect.
Score: 0
Accepted Answers:
True