

Unit 12 - Week 9 - Reproducing Kernel Hilbert Space and Regularization

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Assignment 09

The due date for submitting this assignment has passed. As per our records you have not submitted this assignment.

Due on 2019-10-02, 23:59 IST.

Instructions:

- Attempt all questions.
- Submission deadline: 2nd October 2019 23:59 IST
- Solutions to be posted: 3rd October 2019
- Older browsers might show unnecessary vertical bars at the end of math equations.

1) Let V (assume completeness of V) consist of band-limited functions $f : \mathbb{R} \rightarrow \mathbb{R}$ expressible as $f(t) = \frac{1}{2\pi} \int_{-a}^a F(w) \exp(jwt) dw$, where $F(w)$ is square-integrable. Endow V with the inner product $\langle f, g \rangle = \int_{-\infty}^{\infty} f(t)g(t)dt$. Then V is a reproducing kernel Hilbert space (RKHS) with which of the following kernel?

- $K(t, \tau) = \frac{\sin(a(t+\tau))}{\pi(t-\tau)}$
- $K(t, \tau) = \frac{\sin(a(t-\tau))}{\pi(t+\tau)}$
- $K(t, \tau) = \frac{\sin(a(t+\tau))}{\pi(t+\tau)}$
- $K(t, \tau) = \frac{\sin(a(t-\tau))}{\pi(t-\tau)}$

No, the answer is incorrect. Score: 0

Accepted Answers: $K(t, \tau) = \frac{\sin(a(t-\tau))}{\pi(t-\tau)}$

2) A reproducing kernel, if it exists, is unique. 2 points

- True
- False

No, the answer is incorrect. Score: 0

Accepted Answers: True

3) L_2 is defined as set of all f such that $\int_{-\infty}^{\infty} \|f(x)\|^2 dx < \infty$. Which of the following statements is true? 2 points

- L_2 space is neither a Hilbert space nor an RKHS.
- L_2 space is a Hilbert space but not an RKHS.
- L_2 space is not a Hilbert space but an RKHS.
- L_2 space is a Hilbert space and an RKHS.

No, the answer is incorrect. Score: 0

Accepted Answers: L_2 space is a Hilbert space but not an RKHS.

4) Let $K(i, j) = \delta_{ij}$ where δ_{ij} is defined as follows: 3 points

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j. \end{cases}$$

Then, K is the reproducing kernel of \mathcal{H} as K satisfies

- $\forall j \in \mathbb{N}, K(\cdot, j) = (0, 0, \dots, 1, 0, 0) \in \mathcal{H}$ (1 at the j -th place) and $\forall j \in \mathbb{N}, \forall x = (x_i)_{i \in \mathbb{N}} \in \mathcal{H}, \langle x, K(\cdot, j) \rangle_{\mathcal{H}} = \sum_{i \in \mathbb{N}} x_i \delta_{ij} = x_j$.
- $\forall j \in \mathbb{N}, K(\cdot, j) = (0, 0, \dots, 1, 0, 0) \notin \mathcal{H}$ (1 at the j -th place) and $\forall j \in \mathbb{N}, \forall x = (x_i)_{i \in \mathbb{N}} \in \mathcal{H}, \langle x, K(\cdot, j) \rangle_{\mathcal{H}} = \sum_{i \in \mathbb{N}} x_i \delta_{ij} = x_j$.
- $\forall j \in \mathbb{N}, K(\cdot, j) = (0, 0, \dots, 1, 0, 0) \notin \mathcal{H}$ (1 at the j -th place) and $\forall j \in \mathbb{N}, \forall x = (x_i)_{i \in \mathbb{N}} \in \mathcal{H}, \langle x, K(\cdot, j) \rangle_{\mathcal{H}} = \sum_{i \in \mathbb{N}} x_i \delta_{ij} \neq x_j$.
- $\forall j \in \mathbb{N}, K(\cdot, j) = (0, 0, \dots, 1, 0, 0) \in \mathcal{H}$ (1 at the j -th place) and $\forall j \in \mathbb{N}, \forall x = (x_i)_{i \in \mathbb{N}} \in \mathcal{H}, \langle x, K(\cdot, j) \rangle_{\mathcal{H}} = \sum_{i \in \mathbb{N}} x_i \delta_{ij} \neq x_j$.

No, the answer is incorrect. Score: 0

Accepted Answers: $\forall j \in \mathbb{N}, K(\cdot, j) = (0, 0, \dots, 1, 0, 0) \in \mathcal{H}$ (1 at the j -th place) and $\forall j \in \mathbb{N}, \forall x = (x_i)_{i \in \mathbb{N}} \in \mathcal{H}, \langle x, K(\cdot, j) \rangle_{\mathcal{H}} = \sum_{i \in \mathbb{N}} x_i \delta_{ij} = x_j$.

5) (True/False): Classifying data points linearly using perceptron is a well-posed problem in Hadamard sense. 2 points

- True
- False

No, the answer is incorrect. Score: 0

Accepted Answers: False

6) Let (e_1, e_2, \dots, e_n) be an orthonormal basis in \mathcal{H} . Define $K(x, y) = \sum_{i=1}^n e_i(x)\bar{e}_i(y)$ (\bar{e}_i is the complex conjugate of e_i). Then for any y in E (E is a non-empty abstract space), $K(\cdot, y) = \sum_{i=1}^n \bar{e}_i(y)e_i(\cdot)$ belongs to \mathcal{H} . For any function $\phi(\cdot) = \sum_{i=1}^n \lambda_i e_i(\cdot)$ in \mathcal{H} , compute $\langle \phi, K(\cdot, y) \rangle_{\mathcal{H}}$ 2 points

- $\phi(y)$
- $K(y)$
- $e_i(y)$
- $K(x, y)$

No, the answer is incorrect. Score: 0

Accepted Answers: $\phi(y)$

7) In continuation with question 6, any finite dimensional Hilbert space of functions has a reproducing kernel. 2 points

- True
- False

No, the answer is incorrect. Score: 0

Accepted Answers: True

8) Let \mathcal{H} be a RKHS and $k(x, \cdot)$ be a Mercer kernel. Then any function $f(\cdot) \in \mathcal{H}$ can be decomposed into sum of two components, say $f_{||}$ and f_{\perp} . $f_{||}$ is in the span of the kernel functions $k(x_1, \cdot), k(x_2, \cdot), \dots, k(x_l, \cdot)$ and f_{\perp} is orthogonal to the span of the kernel functions. Compute $f(x_j)$ 2 points

- $f(x_j) = \sum_{i=1}^l a_i k(x_i, \cdot) + \sum_{j=1}^l f_{\perp}(x_j)$
- $f(x_j) = \sum_j \sum_{i=1}^l a_i k(x_j, x_i) + f_{\perp}(x_j)$
- $f(x_j) = \sum_{j=1}^l \sum_{i=1}^l a_i k(x_j, x_i)$
- $f(x_j) = \sum_{i=1}^l a_i k(x_i, x_j)$

No, the answer is incorrect. Score: 0

Accepted Answers: $f(x_j) = \sum_{i=1}^l a_i k(x_i, x_j)$

9) Let \mathcal{H} be a Hilbert space. Given any positive semi-definite kernel k , then we can construct a RKHS using $k(x, \cdot)$. 2 points

- True
- False

No, the answer is incorrect. Score: 0

Accepted Answers: True