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reviewer4@nptel.iitm.ac.in ▾

Courses » Mathematical Methods and Techniques in Signal Processing

Announcements **Course** Ask a Question Progress FAQ

Week 3 - Vector Spaces - III and Signal Geometry

Register for Certification exam

Course outline

How to access the portal

Week 0 - Background and Prerequisites

Week 1 - Introduction to Signal Processing, State Space Representation and Vector Spaces - I

Week 2 - Vector Spaces - II

Week 3 - Vector Spaces - III and Signal Geometry

- Linear independence of orthogonal vectors
- Hilbert space and linear transformation
- Gram Schmidt orthonormalization
- Linear

Assignment 03

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-02-20, 23:59 IST.

Instructions:

1. **Attempt all questions.**
2. **Submission deadline**: 20th February 2019 23:55 IST
3. **Solutions to be posted**: 21st February 2019
4. Older browsers might show **unnecessary vertical bars** at the end of math equations

1) (True/False) If f and g are orthonormal vectors in $\mathbb{R}^n$, then the vectors $u = f + g$ and $v = f - g$ are also orthonormal vectors in $\mathbb{R}^n$. ***1 point***

True

False

No, the answer is incorrect.
Score: 0

Accepted Answers:
False

2) (True/False) Consider a space S spanned by M orthogonal signals. Consider another space S' which is spanned by the negatives of the M orthogonal signals. Then, dimension of the signal space spanned by $S \cup S'$ remains unchanged. ***1 point***

True

False

No, the answer is incorrect.
Score: 0

Accepted Answers:
True

3) (True/False) $\mathrm{L_p}$ space is a Hilbert space as well as a Banach space. ***1 point***

4) Let $u_1 = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ and $y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Let $\{u_1, u_2\}$ be a basis for $W = \text{Span}\{u_1, u_2\}$. Write y as the sum of a vector in W and a vector orthogonal to W. ***1.5 points***

$$y = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix}$$

$$y = \begin{bmatrix} -2/5 \\ 2 \\ 1/5 \end{bmatrix} + \begin{bmatrix} 7/5 \\ 0 \\ 14/5 \end{bmatrix}$$

$$y = \begin{bmatrix} -2/5 \\ 0 \\ 14/5 \end{bmatrix} + \begin{bmatrix} 7/5 \\ 2 \\ 1/5 \end{bmatrix}$$

$$y = \begin{bmatrix} -2/3 \\ 2/3 \\ 14/3 \end{bmatrix} + \begin{bmatrix} 5/3 \\ 4/3 \\ -5/3 \end{bmatrix}$$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$$y = \begin{bmatrix} -2/5 \\ 2 \\ 1/5 \end{bmatrix} + \begin{bmatrix} 7/5 \\ 0 \\ 14/5 \end{bmatrix}$$

5) Suppose $\{\phi_i\}_{i=1}^{n}$ be a set of orthonormal basis for a vector space V. Then for any two vectors x and y belonging to V, ***1.5 points***

$$\langle x, y\rangle = \sum_{i=1}^{n} \langle x + y, \phi_i\rangle$$

$$\langle x, y\rangle = \sum_{i=1}^{n} \langle x, \phi_i\rangle \overline{\langle y, \phi_i\rangle}$$

$$\langle x, y\rangle = \langle x, \phi_i\rangle \overline{\langle y, \phi_i\rangle}$$

$$\langle x, y\rangle = \sum_{i=1}^{n} \sum_{j=1}^{n} \langle x, \phi_i\rangle \langle y, \phi_j\rangle$$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$$\langle x, y\rangle = \sum_{i=1}^{n} \langle x, \phi_i\rangle \overline{\langle y, \phi_i\rangle}$$

6) Given the following set of functions $\{1, e^{jt}, e^{\frac{jt}{4}}\}$ defined over $[-\pi, \pi]$. Which of the following statements are true: ***2 points***

1 and $e^{\frac{jt}{4}}$ are orthogonal.

- [ ] 1 and e^{jt} are orthogonal.
- [ ] e^{jt} and $e^{\frac{jt}{4}}$ are orthogonal.
- [ ] All of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:
1 and e^{jt} are orthogonal.

7) Consider the following linearly independent vectors **2 points**

in $\mathbb{R}^3$: $u_1 = \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}$, $u_2 = \begin{pmatrix} 1 \\ 1 \\ -7 \end{pmatrix}$, $u_3 = \begin{pmatrix} 3 \\ -1 \\ 8 \end{pmatrix}$. Which of the following represents an ordered orthonormal basis for $\mathbb{R}^3$

- [ ] $\mathbf{q_1} = \begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{pmatrix}$, $\mathbf{q_2} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix}$, $\mathbf{q_3} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$
- [ ] $\mathbf{q_1} = \begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}$, $\mathbf{q_2} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$, $\mathbf{q_3} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix}$
- [ ] $\mathbf{q_1} = \begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{pmatrix}$, $\mathbf{q_2} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$, $\mathbf{q_3} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix}$
- [ ] $\mathbf{q_1} = \begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{pmatrix}$, $\mathbf{q_2} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix}$, $\mathbf{q_3} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\mathbf{q_1} = \begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{pmatrix}$, $\mathbf{q_2} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix}$, $\mathbf{q_3} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$

8) (True/False). Let $X = l_2[-\pi, \pi]$. **2 points**

Let $S_1 = \text{span}(\{1,\ \cos(t),\ \cos(2t), \ldots\})$ and $S_2 = \text{span}(\{\sin(t),\ \sin(2t), \ldots\})$. Then, the dimension of the sum of spaces $\dim(S_1 + S_2) = \dim(S_1) + \dim(S_2)$.

- [ ] True

False

No, the answer is incorrect.
Score: 0

Accepted Answers:
True

9) (True/False) For a matrix $A \in \mathbb{R}^{m \times n}$, we denote the null space of A^{T} as $\mathcal{N}_{A^{\mathrm{T}}}$ and null space of AA^{T} as $\mathcal{N}_{AA^{\mathrm{T}}}$. Then $\mathcal{N}_{A^{\mathrm{T}}} = \mathcal{N}_{AA^{\mathrm{T}}}$. ***2 points***

True
False

No, the answer is incorrect.
Score: 0

Accepted Answers:
True

10)Consider a set of linearly independent functions $\{p_1(x) = 1,\ p_2(x) = x,\ p_3(x) = x^2\}$ defined over $[-1, 1]$. An orthonormal basis for these functions is given by ***3 points***

$\mathbf{q_1} = \frac{1}{\sqrt{2}},\ \mathbf{q_2} = \sqrt{\frac{2}{3}}x,\ \mathbf{q_3} = \sqrt{\frac{8}{45}}\left(x^2 - \frac{1}{3}\right)$

$\mathbf{q_1} = 1,\ \mathbf{q_2} = \sqrt{\frac{3}{2}}x,\ \mathbf{q_3} = \sqrt{\frac{45}{8}}\left(x^2 - \frac{1}{3}\right)$

$\mathbf{q_1} = \frac{1}{\sqrt{2}},\ \mathbf{q_2} = \sqrt{\frac{3}{2}}x,\ \mathbf{q_3} = \sqrt{\frac{45}{8}}\left(x^2 - \frac{1}{3}\right)$

$\mathbf{q_1} = \frac{1}{\sqrt{2}},\ \mathbf{q_2} = \sqrt{\frac{3}{2}}x,\ \mathbf{q_3} = \sqrt{45}\left(x^2 - \frac{1}{3}\right)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\mathbf{q_1} = \frac{1}{\sqrt{2}},\ \mathbf{q_2} = \sqrt{\frac{3}{2}}x,\ \mathbf{q_3} = \sqrt{\frac{45}{8}}\left(x^2 - \frac{1}{3}\right)$

11)Consider the following signals: ***3 points***

$$f_1(t) = \begin{cases} 0.5 & \text{for } 0 \leq t \leq 2 \\ -0.5 & \text{for } 2 \leq t \leq 4 \end{cases}$$

$$f_2(t) = \begin{cases} 0.5 & \text{for } 0 \leq t \leq 4 \\ 0 & \text{for } t \geq 4 \end{cases}$$

$$f_3(t) = \begin{cases} 0.5 & \text{for } 0 \leq t \leq 1 \text{ and } 2 \leq t \leq 3 \\ -0.5 & \text{for } 1 \leq t \leq 2 \text{ and } 3 \leq t \leq 4 \end{cases}$$

Consider a signal $x(t)$ defined as below

$$x(t) = \begin{cases} 2 & \text{for } 0 \leq t \leq 1 \\ 3 & \text{for } 1 \leq t \leq 2 \\ -1 & \text{for } 2 \leq t \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

$x(t)$ can be approximated by the signals $f_i(t)$ for $i = 1, 2, 3$ as

$x(t) = f_1(t) + 2f_2(t) - 3f_3(t)$

- $x(t) = 3f_1(t) + 2f_2(t) - f_3(t)$
- $x(t) = f_1(t) + 3f_2(t) - 2f_3(t)$
- $x(t) = 2f_1(t) + 2f_2(t) - 4f_3(t)$
- Not possible to represent $x(t)$ with given $f_i(t)$ for $i = 1, 2, 3$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
$x(t) = 3f_1(t) + 2f_2(t) - f_3(t)$

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