

## Week 8 -

Multirate
Systems - IV

## Week 9 -

Wavelets - I

- Introduction to wavelets
- Multiresolution analysis and properties
- The Haar wavelet
- Structure of subspaces in MRA

Haar decomposition 1

- Haar decomposition 2

Quiz : Assignment 09

Assignment 9 Solutions

Week 10 -
Wavelets - II and
Continuity of
Functions

Week 11 -
Fourier Series - I

Week 12 -
Fourier Series -
II and KL
Transform

Interaction
Session

$$
\begin{aligned}
& \phi\left(2^{j-1} t\right)=\phi\left(2^{j} t\right)-\phi\left(2^{j} t-1\right) \\
& \psi\left(2^{j-1} t\right)=\phi\left(2^{j} t\right)-\phi\left(2^{j} t-1\right) \\
& \phi(t-3)=\phi(2 t-6)+\phi(2 t-7) \\
& \square \\
& \psi(t-2)=\phi(2 t-4)+\phi(2 t-5)
\end{aligned}
$$

No, the answer is incorrect.
Score: 0

$$
\begin{aligned}
& \text { Accepted Answers: } \\
& \begin{array}{l}
\psi\left(2^{j-1} t\right)=\phi\left(2^{j} t\right)-\phi\left(2^{j} t-1\right) \\
\phi(t-3)=\phi(2 t-6)+\phi(2 t-7)
\end{array}
\end{aligned}
$$

4) The $j^{\text {th }}$ scale approximation of a signal $f(t)$ using Haar wavelets can be written in two 1.5 points forms as
$f_{j}(t)=\sum_{k=-\infty}^{\infty} a_{k}^{(j)} \phi\left(2^{j-1} t-k\right)+\sum_{k=-\infty}^{\infty} b_{k}^{(j)} \psi\left(2^{j-1} t-k\right)$
$f_{j}(t)=\sum_{k=-\infty}^{\infty} c_{k}^{(j)} \phi\left(2^{j} t-k\right)$
Choose the correct statements.

$$
\begin{aligned}
& a_{k}^{(j)}=\frac{1}{2}\left(c_{2 k}^{(j)}+c_{2 k+1}^{(j)}\right) \\
& \square \\
& a_{k}^{(j)}=\frac{1}{2}\left(c_{2 k}^{(j)}-c_{2 k+1}^{(j)}\right) \\
& \square \\
& b_{k}^{(j)}=\frac{1}{2}\left(c_{2 k}^{(j)}-c_{2 k+1}^{(j)}\right) \\
& \square \\
& b_{k}^{(j)}=\frac{1}{2}\left(c_{2 k-1}^{(j)}-c_{2 k}^{(j)}\right)
\end{aligned}
$$

No, the answer is incorrect.
Score: 0
Accepted Answers:

$$
\begin{aligned}
& a_{k}^{(j)}=\frac{1}{2}\left(c_{2 k}^{(j)}+c_{2 k+1}^{(j)}\right) \\
& b_{k}^{(j)}=\frac{1}{2}\left(c_{2 k}^{(j)}-c_{2 k+1}^{(j)}\right)
\end{aligned}
$$

5) Given
$f(t)= \begin{cases}1, & 0 \leq t<0.25 \\ 0, & 0.25 \leq t<0.5 \\ 3, & 0.5 \leq t<0.75 \\ -2, & 0.75 \leq t<1\end{cases}$
The Haar wavelet decomposition of the signal $f(t)$ is given by

$$
\begin{aligned}
& f(t)=\frac{1}{2} \phi(t)+2 \psi(t)+\frac{1}{2} \psi(2 t)+\frac{5}{2} \psi(2 t-1) \\
& f(t)=\phi(t)+\frac{1}{2} \psi(2 t)+\frac{3}{2} \psi(2 t-1)
\end{aligned}
$$

$$
\begin{aligned}
& f(t)=\frac{1}{2} \phi(t)+\frac{1}{2} \psi(2 t)+\frac{5}{2} \psi(2 t-1) \\
& f(t)=\frac{1}{2} \phi(t)+\psi(t)+\frac{1}{2} \psi(2 t)+\frac{5}{2} \psi(2 t-1)
\end{aligned}
$$

No, the answer is incorrect.
Score: 0
Accepted Answers:

$$
f(t)=\frac{1}{2} \phi(t)+\frac{1}{2} \psi(2 t)+\frac{5}{2} \psi(2 t-1)
$$

6) In question 5 , what is the signal dimension of $f(t)$ in a space spanned by Haar scaling function $\phi(t)$, Haar wavelets $\psi\left(2^{i} t\right)$ and their time shifted versions?

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 3
7) In question 5, let $\hat{f}(t)$ be the signal obtained if the subspace corresponding to the details 1 point at the highest resolution is nulled out. What is $\hat{f}(t)$ ?

$$
\begin{aligned}
& \hat{f}(t)=\frac{1}{2} \phi(t) \\
& \hat{f}(t)=\frac{1}{2} \phi(t)+\psi(t) \\
& \hat{f}(t)=\frac{1}{2} \phi(t)+\frac{1}{2} \psi(2 t) \\
& \hat{f}(t)=\phi(t)+\frac{1}{2} \psi(2 t)
\end{aligned}
$$

No, the answer is incorrect.
Score: 0
Accepted Answers:

$$
\hat{f}(t)=\frac{1}{2} \phi(t)
$$

8) From questions 5 and $7, \mathbf{x} \%$ of the energy is lost in representing the signal $f(t)$ as $\hat{f}(t)$. What is the value of $\mathbf{x}$ ?(Round it to nearest integer).

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 93
9) We define a sequence of spaces
as $\mathcal{V}_{k}=\operatorname{Span}\left(\left\{\sin \left(2 \pi 2^{k} t\right), \cos \left(2 \pi 2^{k} t\right)\right\}\right)$ for $k=\cdots,-1,0,1, \cdots$. Choose the correct statements.

The spaces $\mathcal{V}_{k}, k \in \mathbb{Z}$ satisfy the nesting property.

$$
\begin{aligned}
& \bigcap_{k=-\infty}^{k=\infty} \mathcal{V}_{k}=\{0\} . \\
& \square \\
& \text { The spaces } \mathcal{V}_{k}, k \in \mathbb{Z} \text { satisfy the scaling property. } \\
& \square \\
& \text { None of the above. }
\end{aligned}
$$

No, the answer is incorrect.
Score: 0
Accepted Answers:

$$
\bigcap_{k=-\infty}^{k=\infty} \mathcal{V}_{k}=\{0\}
$$

The spaces $\mathcal{V}_{k}, k \in \mathbb{Z}$ satisfy the scaling property.
10Any function $f(t) \in L^{2}(\mathbb{R})$ can be approximated using $j^{\text {th }}$ scale of Haar scaling 1.5 poinco function as $f_{j}(t)=\sum_{k=-\infty}^{\infty} a_{k}^{(j)} \phi\left(2^{j} t-k\right)$. Which of the following expression is used to calculate the coefficients $a_{k}^{(j)}$ ?

$$
\begin{aligned}
& a_{k}^{(j)}=\int_{2^{-j} k}^{2^{-j}(k+1)} f(t) d t \\
& a_{k}^{(j)}=\frac{\left\langle f(t), \phi\left(2^{j} t-k\right)\right\rangle}{\left\langle\phi\left(2^{j} t-k\right), \phi\left(2^{j} t-k\right)\right\rangle} \\
& a_{k}^{(j)}=\left\langle f(t), \phi\left(2^{j} t-k\right)\right\rangle \\
& a_{k}^{(j)}=\frac{1}{2^{-j}} \int_{2^{-j} k}^{2^{-j}(k+1)} f(t) d t
\end{aligned}
$$

No, the answer is incorrect.
Score: 0
Accepted Answers:

$$
\begin{aligned}
& a_{k}^{(j)}=\frac{\left\langle f(t), \phi\left(2^{j} t-k\right)\right\rangle}{\left\langle\phi\left(2^{j} t-k\right), \phi\left(2^{j} t-k\right)\right\rangle} \\
& a_{k}^{(j)}=\frac{1}{2^{-j}} \int_{2^{-j} k}^{2^{-j}(k+1)} f(t) d t
\end{aligned}
$$

11)The signal $f(t)=3 t+4$ is approximated using the $j^{\text {th }}$ scale approximation of Haar 2 points wavelets given by $f_{j}(t)=\sum_{k=-\infty}^{\infty} a_{k}^{(j)} \phi\left(2^{j} t-k\right)$. Choose the correct statements.

$$
\begin{aligned}
& a_{k}^{(j)}=2^{-j}\left(k+\frac{1}{2}\right)+2 \\
& a_{k}^{(j)}=3 \times 2^{-j}\left(k+\frac{1}{2}\right)+4 \\
& a_{k}^{(j)}=3 \times 2^{-j}(k+1)+2 \\
& a_{k}^{(j)}=-2^{-j}\left(k+\frac{1}{4}\right)-3
\end{aligned}
$$

No, the answer is incorrect.

Score: 0
Accepted Answers:
$a_{k}^{(j)}=3 \times 2^{-j}\left(k+\frac{1}{2}\right)+4$
12) Using question 11 , what is the value of $8 a_{4}^{(2)}-32 a_{3}^{(4)}$ ?96-90
No, the answer is incorrect.
Score: 0
Accepted Answers:
-90

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