## Introduction to Time-Varying Electrical Networks : Week 7

## **Problem 1**



Figure 1: Time-varying RC filter for problem 1.

Fig. 1 shows an RC network with a periodically time-varying resistor. The capacitor C = 1 F. The conductance varies from 0 to  $2\pi 1000$  S as shown in the figure. Write a MATLAB program to determine the harmonic transfer functions of this network. Assume that g(t) has a rise- and fall-times of 0.1 s and an average value that is half its peak value. Denoting the maximum number of harmonics at any node by K, so that there are (2K + 1) sinusoids at any node, your code should be able to accommodate a user-specified K. Plot  $|H_k(j2\pi f)|$  for  $k = 0, \pm 1$ , for K = 128, 256, 512. For uniformity, the range of the y-axis must be 0-1, and the x-axis from -3 to 3 Hz, with increments in f chosen to be 0.05. What do you notice as you change K? Run sanity checks for the values of the harmonic transfer functions at frequencies 1, 2, 3 Hz.

Suggestion : When you invert the **G** matrix in MATLAB to solve  $\mathbf{GV} = \mathbf{I}$ , do *not* use  $\forall = in\forall (G) * I$ . Turns out that matrix inversion is a very computationally intensive process – remember that you are inverting a  $3000 \times 3000$  matrix. Rather, use  $\forall = G \setminus I$ , which is much quicker. Since I told you not to do something, I am sure you will definitely do it. See for yourself how much quicker the  $\forall = G \setminus I$  is with respect to explicitly computing the inverse.

## Problem 5

Fig. 2 shows an RC network with a periodically-operated ideal switch. The switch is closed when s(t) is high and open when it is low. s(t) has a frequency of 1 Hz and a 25% duty cycle. The capacitor C = 1 F. RC = 10 s. The output is the voltage at the output of the capacitor  $v_c(t)$ . Analytically determine  $H_{-k}(j2\pi k)$  for 1, 2 (use the fact that  $RC \gg T_s/4$  to advantage).



Figure 2: Switched RC filter for problem 2.

Next, use the code you developed in the previous problem to compute  $H_{-k}(j2\pi k)$  for 1, 2. Use K = 512, and a frequency resolution of 0.01 Hz. Plot  $H_{-k}(j2\pi f)$  for 1, 2 for f between 0 and 3 Hz at intervals of every 0.01 Hz. What do you notice? For ease of computation, assume that the rise- and fall-times of s(t) are 1% of its period, and that the average of s(t) is 25% of its peak value.