# Introduction to Time-Varying Electrical Networks : Week 7 

## Problem 1



Figure 1: Time-varying RC filter for problem 1.

Fig. 1 shows an RC network with a periodically time-varying resistor. The capacitor $C=1 \mathrm{~F}$. The conductance varies from 0 to $2 \pi 1000 \mathrm{~S}$ as shown in the figure. Write a MATLAB program to determine the harmonic transfer functions of this network. Assume that $g(t)$ has a rise- and fall-times of 0.1 s and an average value that is half its peak value. Denoting the maximum number of harmonics at any node by $K$, so that there are $(2 K+1)$ sinusoids at any node, your code should be able to accommodate a user-specified $K$. Plot $\left|H_{k}(j 2 \pi f)\right|$ for $k=0, \pm 1$, for $K=128,256,512$. For uniformity, the range of the $y$-axis must be $0-1$, and the $x$-axis from -3 to 3 Hz , with increments in $f$ chosen to be 0.05 . What do you notice as you change $K$ ? Run sanity checks for the values of the harmonic transfer functions at frequencies $1,2,3 \mathrm{~Hz}$.

Suggestion : When you invert the G matrix in MATLAB to solve $\mathbf{G V}=\mathbf{I}$, do not use $\mathrm{V}=\operatorname{inv}(\mathrm{G}) * \mathrm{I}$. Turns out that matrix inversion is a very computationally intensive process - remember that you are inverting a $3000 \times 3000$ matrix. Rather, use $V=G \backslash I$, which is much quicker. Since I told you not to do something, I am sure you will definitely do it. See for yourself how much quicker the $V=G \backslash I$ is with respect to explicitly computing the inverse.

## Problem 5

Fig. 2 shows an RC network with a periodically-operated ideal switch. The switch is closed when $s(t)$ is high and open when it is low. $s(t)$ has a frequency of 1 Hz and a $25 \%$ duty cycle. The capacitor $C=1 \mathrm{~F} . R C=10 \mathrm{~s}$. The output is the voltage at the output of the capacitor $v_{c}(t)$. Analytically determine $H_{-k}(j 2 \pi k)$ for 1,2 (use the fact that $R C \gg T_{s} / 4$ to advantage).


Figure 2: Switched RC filter for problem 2.

Next, use the code you developed in the previous problem to compute $H_{-k}(j 2 \pi k)$ for 1,2 . Use $K=512$, and a frequency resolution of 0.01 Hz . Plot $H_{-k}(j 2 \pi f)$ for 1,2 for $f$ between 0 and 3 Hz at intervals of every 0.01 Hz . What do you notice? For ease of computation, assume that the rise- and fall-times of $s(t)$ are $1 \%$ of its period, and that the average of $s(t)$ is $25 \%$ of its peak value.

