Introduction to Time-Varying Electrical Networks: Assignment Week 5

Problem 1

In class, we denoted the impulse response of a linear time-varying system by $h(t,\tau)$. Here t is time at which the output is observed, while the input impulse is applied at an instant $(t-\tau)$. An alternative notation for the impulse response, that is also in use, is $\hat{h}(t,\tau)$ where

- t denotes the time of observation (as before).
- τ denotes the time at which the input impulse is applied.

The aim of this problem is to develop the results in the class using this alternative notation.

a. What is/are the constraint(s) that $\hat{h}(t,\tau)$ should satisfy if the LTV system is causal?

b. How is $h(t,\tau)$ related to $\hat{h}(t,\tau)$?

c. Determine the response of the LTV system to a complex exponential $e^{j2\pi ft}$ in terms of $\hat{h}(t,\tau)$, and thereby express the time-varying frequency response in terms of $\hat{h}(t,\tau)$.

d. What is the constraint that must be satisfied by $\hat{h}(t,\tau)$ if the system is also periodically time-varying?

Problem 2

This problem involves the use of a SPICE simulator. If you have a favorite, use it. Else, I recommend LTSPICE, which is free and versatile.

Create a first-order time-varying RC filter as shown in the figure. The resistor$R(t)$ varies time from 0.1Ω to 10Ω as shown (the rise- and fall-times are 0.1 s). Use the technique discussed in class to plot the magnitude and phase responses (in degrees) of the time-varying filter as a function of time from 0-20 s. Suggest some sanity checks to be convinced of the correctness of your plots.

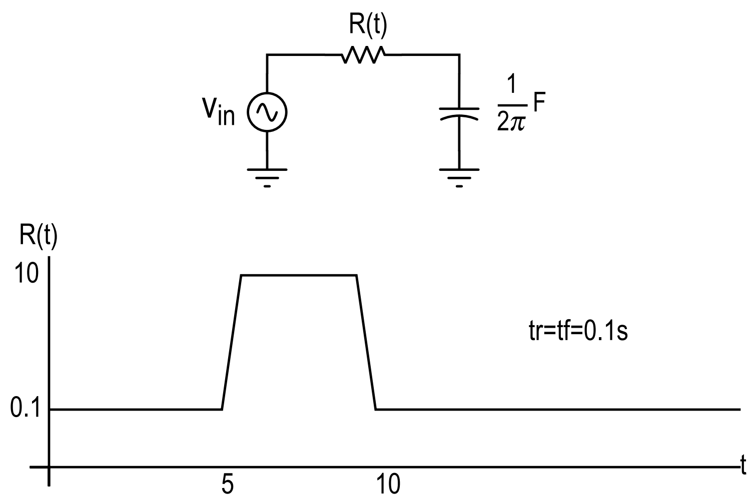

Figure 1: Time-varying circuit for problem 2.