Introduction to Time-Varying Electrical Networks
Assignment for Week 1

Problem 1

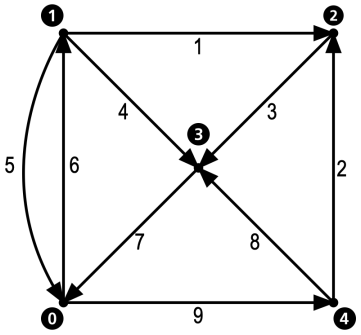

Figure 1: Directed graph for problem 1.

Fig. 1 shows the directed graph of a network, with the nodes and branches marked. Choose node 0 to be the reference (datum) and determine the incidence matrix of this graph.

Problem 2

Consider two topologically identical networks $\mathcal{N}$ and $\hat{\mathcal{N}}$. We denote the branch currents and branch voltages in these networks (as a function of time) by $v_k(t), i_k(t)$ and $\hat{v}_k(t), \hat{i}_k(t)$ respectively. Further, let t_1 and t_2 be two different time instants. Determine

a. $\sum_k v_k(t_1) i_k(t_2)$

b. $\sum_k v_k(t_1) \hat{i}_k(t_2)$

c. $\sum_k \hat{v}_k(t_1) \left.\frac{di_k}{dt}\right|_{t=t_2}$

Problem 3

Let $\mathcal{N}$ be a two-port made of arbitrary interconnections of linear resistors (i.e., each resistor satisfies Ohm's law: $v = iR$). Consider

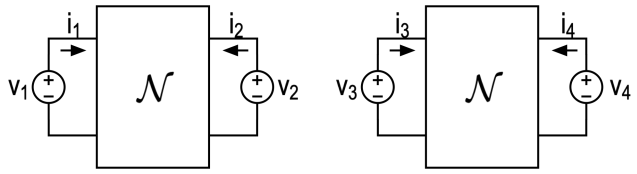

Figure 2: Network for problem 3.

the following two experiments:

Experiment 1. Drive $\mathcal{N}$ with two voltage sources v_1 and v_2 and measure the resulting port currents i_1 and i_2 respectively.

Experiment 2. Drive $\mathcal{N}$ with two voltage sources v_3 and v_4 and measure the resulting port currents i_3 and i_4 respectively.

Prove that the two sets of measurements are related as follows:

$v_1 i_3 + v_2 i_4 = v_3 i_1 + v_4 i_2$

Problem 4

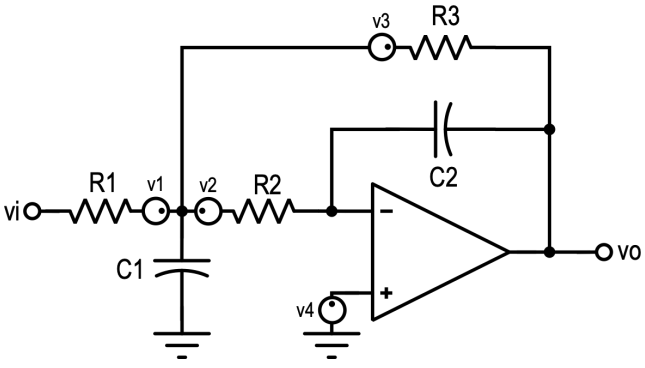

Figure 3: Network for problem 4.

In the network of Fig. 3, the opamp is ideal. Determine the transfer functions from v_i, v_1, v_2, v_3 to v_o. The positive terminals of the voltage sources are marked with a dot.

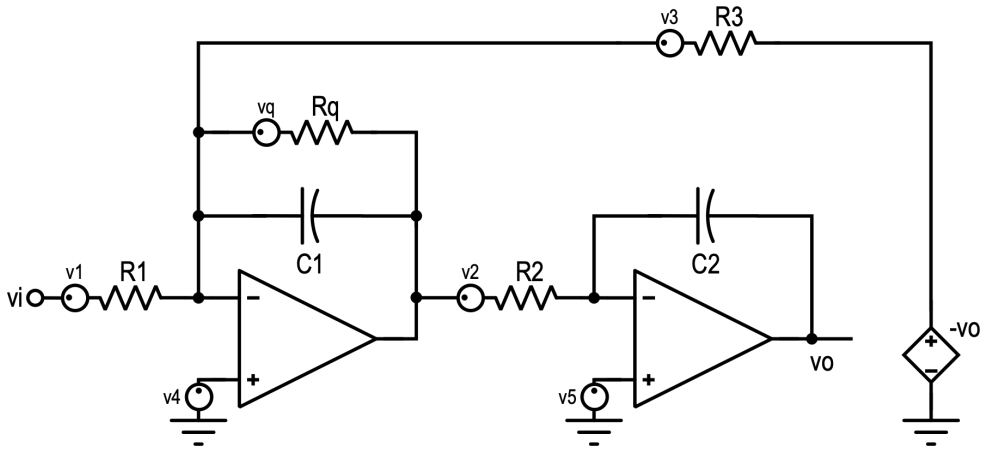

Figure 4: Network for problem 5.

Problem 5

In the network of Fig. , the opamps are ideal. Determine the transfer functions from $v_i, v_1, \cdots, v_5, v_q$, to v_o. The positive terminals of the voltage sources are marked with a dot.