Introduction to Nonlinear Dynamics
Indian Institute of Technology Madras
NPTEL September 2015

Assignment 4

Due: October 25, 2015, 23:30 (IST)

1 Examples and Definitions

1) For each of the following vector fields, find and classify all the fixed points, and sketch the phase portraits on the circle.
   a) $\dot{\theta} = 1 + 2\cos\theta$
   b) $\dot{\theta} = \sin\theta + \cos\theta$

2 Uniform Oscillator

1) [*The clock problem*] At 12 : 00, the hour hand and minute hand of a clock are perfectly aligned. When is the *next* time they will be aligned? (Solve the problem by the methods of this section, and also by some alternative approach of your choice.)

3 Nonuniform Oscillator

1) For the following systems, draw the phase portrait as a function of the control parameter μ. Classify the bifurcations that occur as μ varies, and find all the bifurcation values of μ.
   a) $\dot{\theta} = \mu\sin\theta - \sin 2\theta$
   b) $\dot{\theta} = \mu + \cos\theta + \cos 2\theta$

4 Linear Systems

1) Consider the system

$$\dot{x} = 4x - y,$$
$$\dot{y} = 2x + y.$$

a) Write the system in the form $\dot{x} = Ax$.

b) Find the characteristic polynomial.

c) Find the eigenvalues and eigenvectors.

d) Classify the fixed point at the origin.