Assignment 3

Due: October 15, 2015, 23:30 (IST)

1 Saddle-Node bifurcation

1) Consider the system $\dot{x} = 1 + rx + x^2$.

    a) Sketch the qualitatively different vector fields that occur as r is varied.

    b) Determine the critical value of r at which a saddle-node bifurcation occurs.

2 Transcritical bifurcation

1) Show that the system $\dot{x} = x(r - e^x)$ undergoes a transcritical bifurcation. Determine the value of r at which the bifurcation occurs and sketch the bifurcation diagram of fixed points x^* vs r.

3 Pitchfork bifurcation

1) Consider the following system $\dot{x} = rx - 4x^3$.

    a) Sketch the qualitatively different vector fields that occur as r is varied.

    b) Determine the critical value of r at which a pitchfork bifurcation occurs.

    c) Classify the bifurcation as supercritical or subcritical.

4 Identify the bifurcation

1) In each of the following systems, determine the value of r at which a bifurcation occurs, and classify the bifurcation as saddle-node, transcritical, supercritical pitchfork or subcritical pitchfork.

    a) $\dot{x} = 5 - re^{-x^2}$

    b) $\dot{x} = r - 3x^2$