Introduction to Nonlinear Dynamics
Indian Institute of Technology Madras
NPTEL September 2015

Assignment 2

Due: October 8, 2015, 23:30 (IST)

1 Existence and uniqueness

1) A particle travels on the half-line $x \geq 0$ with a velocity given by $\dot{x} = -x^c$, where c is real and constant.
   a) Find all values of c such that the origin $x = 0$ is a stable fixed point.
   b) Now assume that c is chosen such that $x = 0$ is stable. Can the particle ever reach the origin in a *finite* time? Specifically, how long does it take for the particle to travel from $x = 1$ to $x = 0$, as a function of c?

2 Impossibility of oscillations

1) [*No periodic solutions to* $\dot{x} = f(x)$] Here's an analytical proof that periodic solutions are impossible for a vector field on a line. Suppose on the contrary that $x(t)$ is a nontrivial periodic solution, *i.e.*, $x(t) = x(t+T)$ for some $T > 0$, and $x(t) \neq x(t+s)$ for all $0 < s < T$. Derive a contradiction by considering $\int_t^{t+T} f(x) \frac{dx}{dt} dt$.

3 Potentials

1) For each of the following vector fields, plot the potential function $V(x)$ and identify all the equilibrium points and their stability.
   a) $\dot{x} = x(1 - x)$
   b) $\dot{x} = 2 + \sin x$
   c) $\dot{x} = r + x - x^3$, for various values of r.