Assignment-4

The due date for submitting this assignment has passed.

Due on 2021-02-17, 23:59 IST.

As per our records you have not submitted this assignment.

1) Consider i.i.d. zero-mean Gaussian random variables $x_1, x_2, \cdots, x_n$ of variance σ^2. Let $\bar{\mathbf{x}} = [x_1, x_2, \cdots, x_n]^T$. The variance of the quantity $\bar{\mathbf{a}}^T\bar{\mathbf{x}}$ is *1 point*

- $\sigma^2 \|\bar{\mathbf{a}}\|$
- $\sigma^2(|a_1| + |a_2| + \cdots + |a_n|)$
- $\sigma^2 \|\bar{\mathbf{a}}\|^2$
- $\sigma^2(a_1 + a_2 + \cdots + a_n)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\sigma^2 \|\bar{\mathbf{a}}\|^2$

2) The eigenvalue decomposition of a real symmetric matrix $\mathbf{A}$ is of the form *1 point*

- $\mathbf{U\Lambda U}^{-1}$, where the $\mathbf{\Lambda}$ is an arbitrary diagonal matrix of eigenvalues and $\mathbf{U}$ is an arbitrary matrix of eigenvectors
- $\mathbf{U\Lambda U}^T$, where the $\mathbf{\Lambda}$ is a diagonal matrix of real eigenvalues and $\mathbf{U}$ is a matrix of orthonormal eigenvectors
- $\mathbf{U\Lambda U}^T$, where the $\mathbf{\Lambda}$ is a diagonal matrix of non-negative eigenvalues and $\mathbf{U}$ is a matrix of orthonormal eigenvectors
- $\mathbf{U\Lambda U}^T$, where the $\mathbf{\Lambda}$ is a diagonal matrix of eigenvalues that have magnitude unity and $\mathbf{U}$ is a matrix of orthonormal eigenvectors

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\mathbf{U\Lambda U}^T$, where the $\mathbf{\Lambda}$ is a diagonal matrix of real eigenvalues and $\mathbf{U}$ is a matrix of orthonormal eigenvectors

3) The eigenvectors of a positive semi-definite matrix corresponding to distinct eigenvalues are *1 point*

- Orthogonal and have unit-norm
- Orthogonal but not necessarily unit-norm
- Neither orthogonal nor unit-norm
- Non-orthogonal and have unit-norm

No, the answer is incorrect.
Score: 0

Accepted Answers:
Orthogonal but not necessarily unit-norm

4) The eigenvalues of the matrix below are *1 point*

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$$

- $4, -1$
- $-2, 1$
- $-3, 2$
- $2, -3$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$4, -1$

5) Consider the Gaussian random vector $\bar{\mathbf{x}}$ with mean $\bar{\boldsymbol{\mu}}_x$ and covariance matrix $\mathbf{R}_x$. Consider now $\bar{\mathbf{y}} = \mathbf{A}\bar{\mathbf{x}} + \bar{\mathbf{b}}$. Its mean and covariance matrix are given as *1 point*

- $\mathbf{A}\bar{\boldsymbol{\mu}}_x, \mathbf{AR}_x\mathbf{A}^T$
- $\mathbf{A}\bar{\boldsymbol{\mu}}_x, \mathbf{AR}_x\mathbf{A}^T + \bar{\mathbf{b}}\bar{\mathbf{b}}^T$
- $\mathbf{A}\bar{\boldsymbol{\mu}}_x + \bar{\mathbf{b}}, \mathbf{AR}_x\mathbf{A}^T + \bar{\mathbf{b}}\bar{\mathbf{b}}^T$
- $\mathbf{A}\bar{\boldsymbol{\mu}}_x + \bar{\mathbf{b}}, \mathbf{AR}_x\mathbf{A}^T$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\mathbf{A}\bar{\boldsymbol{\mu}}_x + \bar{\mathbf{b}}, \mathbf{AR}_x\mathbf{A}^T$

6) Consider a 2 dimensional random column vector $\bar{\mathbf{x}}$ that has the multi-variate Gaussian distribution with mean $\bar{\boldsymbol{\mu}}_x = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$ and covariance matrix $\mathbf{\Sigma}_x = \begin{bmatrix} 6 & 0 \\ 0 & 2 \end{bmatrix}$. Consider the vector $\bar{\mathbf{y}}$ defined as *1 point*

$$\bar{\mathbf{y}} = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}\bar{\mathbf{x}} + \begin{bmatrix} -2 \\ 3 \end{bmatrix}.$$

What is the mean $\bar{\boldsymbol{\mu}}_y$ and covariance matrix $\mathbf{\Sigma}_y$ of $\bar{\mathbf{y}}$?

- $\bar{\boldsymbol{\mu}}_y = \begin{bmatrix} -12 \\ -19 \end{bmatrix}, \mathbf{\Sigma}_y = \begin{bmatrix} 14 & 34 \\ 34 & 86 \end{bmatrix}$
- $\bar{\boldsymbol{\mu}}_y = \begin{bmatrix} -12 \\ -19 \end{bmatrix}, \mathbf{\Sigma}_y = \begin{bmatrix} 12 & 18 \\ 18 & 48 \end{bmatrix}$
- $\bar{\boldsymbol{\mu}}_y = \begin{bmatrix} -6 \\ -15 \end{bmatrix}, \mathbf{\Sigma}_y = \begin{bmatrix} 14 & 34 \\ 34 & 86 \end{bmatrix}$
- $\bar{\boldsymbol{\mu}}_y = \begin{bmatrix} -6 \\ -15 \end{bmatrix}, \mathbf{\Sigma}_y = \begin{bmatrix} 12 & 18 \\ 18 & 48 \end{bmatrix}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\bar{\boldsymbol{\mu}}_y = \begin{bmatrix} -12 \\ -19 \end{bmatrix}, \mathbf{\Sigma}_y = \begin{bmatrix} 14 & 34 \\ 34 & 86 \end{bmatrix}$

7) Consider the Gaussian classification problem with the two classes distributed as *1 point*

$$C_1 \sim N(\bar{\boldsymbol{\mu}}_1, \mathbf{\Sigma})$$
$$C_2 \sim N(\bar{\boldsymbol{\mu}}_2, \mathbf{\Sigma})$$

The classifier for this problem, to classify a new vector $\bar{\mathbf{x}}$, can be formulated as

- Choose C_1 if $(\bar{\boldsymbol{\mu}}_1 - \bar{\boldsymbol{\mu}}_2)^T\mathbf{\Sigma}^{-1}\left(\bar{\mathbf{x}} - \frac{\bar{\boldsymbol{\mu}}_1+\bar{\boldsymbol{\mu}}_2}{2}\right) \leq 0$ and C_2 otherwise
- Choose C_1 if $(\bar{\boldsymbol{\mu}}_1 - \bar{\boldsymbol{\mu}}_2)^T\mathbf{\Sigma}^{-1}\left(\bar{\mathbf{x}} - \frac{\bar{\boldsymbol{\mu}}_1+\bar{\boldsymbol{\mu}}_2}{2}\right) \geq 0$ and C_2 otherwise
- Choose C_1 if $(\bar{\boldsymbol{\mu}}_1 - \bar{\boldsymbol{\mu}}_2)^T\left(\bar{\mathbf{x}} - \frac{\bar{\boldsymbol{\mu}}_1+\bar{\boldsymbol{\mu}}_2}{2}\right) \geq 0$ and C_2 otherwise
- Choose C_1 if $(\bar{\boldsymbol{\mu}}_1 - \bar{\boldsymbol{\mu}}_2)^T\mathbf{\Sigma}\left(\bar{\mathbf{x}} - \frac{\bar{\boldsymbol{\mu}}_1+\bar{\boldsymbol{\mu}}_2}{2}\right) \geq 0$ and C_2 otherwise

No, the answer is incorrect.
Score: 0

Accepted Answers:
Choose C_1 if $(\bar{\boldsymbol{\mu}}_1 - \bar{\boldsymbol{\mu}}_2)^T\mathbf{\Sigma}^{-1}\left(\bar{\mathbf{x}} - \frac{\bar{\boldsymbol{\mu}}_1+\bar{\boldsymbol{\mu}}_2}{2}\right) \geq 0$ and C_2 otherwise

8) Consider the Gaussian classification problem with the two classes C_1, C_2 distributed as *1 point*

$$C_1 \sim N\left(\begin{bmatrix} 2 \\ -4 \end{bmatrix}, \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{8} \end{bmatrix}\right), C_2 \sim N\left(\begin{bmatrix} -4 \\ 2 \end{bmatrix}, \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{8} \end{bmatrix}\right)$$

The classifier for this problem, to classify a new vector $\bar{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, can be formulated as choose C_1 if

- $4x_1 + 6x_2 \geq 2$
- $3x_1 - 2x_2 \geq -1$
- $-2x_1 + 4x_2 \leq 3$
- $x_1 - 2x_2 \geq 1$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$x_1 - 2x_2 \geq 1$

9) Consider a zero-mean random vector to have independent random variables. Its covariance matrix is *1 point*

- Proportional to the identity matrix
- A diagonal matrix but not necessarily proportional to the identity matrix
- An arbitrary positive semi-definite matrix, not necessarily diagonal
- Any arbitrary matrix

No, the answer is incorrect.
Score: 0

Accepted Answers:
A diagonal matrix but not necessarily proportional to the identity matrix

10) The eigenvectors of the matrix below are *1 point*

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$$

- $\begin{bmatrix} 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
- $\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \end{bmatrix}$
- $\begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
- $\begin{bmatrix} -4 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$