Unit 7 - Week 5

Assignment 5

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-03-04, 23:59 IST.

Please refer the formula sheet to solve this assignment. The formula sheet will not be provided in the final exam. Feel free to ask questions on the discussion forum.

GROUP-A

There are 4 questions in this section. Each question contains one mark.

1) A random process $\mathbf{X}(t)$ has following properties: *1 point*

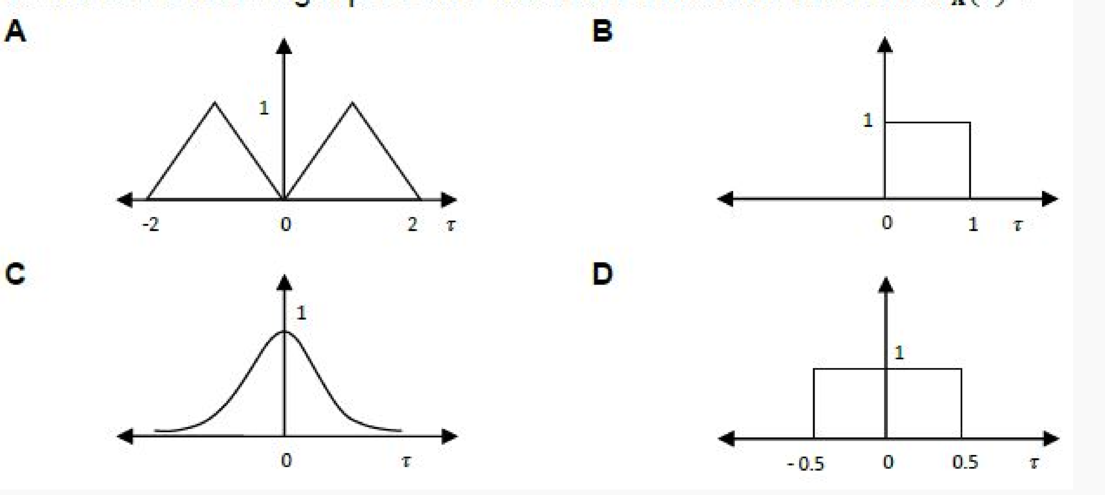
- $E[\mathbf{X}(t)] = a = \text{const}$
- $E[\mathbf{X}(t_1)\mathbf{X}(t_2)] = f(t_1 - t_2)$; ∀ real values of t_1, t_2; $df(x) \rightarrow$ function of x
- $f_{\mathbf{X}(t_1),\mathbf{X}(t_2),\ldots,\mathbf{X}(t_n)}(x_1, x_2, \ldots, x_n) = f_{\mathbf{X}(t_1+d),\mathbf{X}(t_2+d),\ldots,\mathbf{X}(t_n+d)}(x_1, x_2, \ldots, x_n)$, ∀ integer values of n, ∀ real values of d; where d represents time delay

$\mathbf{X}(t)$ belongs to which of the following class of the random processes?

- Strict sense stationary
- Ergodic
- White
- Gaussian

No, the answer is incorrect.
Score: 0
Accepted Answers:
Strict sense stationary

2) Which of the following represent a valid auto-correlation functions $R_{\mathbf{X}}(\tau)$? *1 point*



- A only
- C only
- Both B and D
- Both C and D
- A, C, D
- A, B, C, D

No, the answer is incorrect.
Score: 0
Accepted Answers:
C only

3) A white Gaussian noise process $\mathbf{X}(t)$ is sampled at n different time instants to get n random variables as $\mathbf{X}(t_i), 1 < i < n$. Which of the following are true? *1 point*

A $\mathbf{X}(t_i)$ are correlated jointly Gaussian random variables
B $\mathbf{X}(t_i)$ are jointly Gaussian random variables
C $\mathbf{X}(t_i)$ are statistically independent Gaussian random variables
D All $\mathbf{X}(t_i)$ have identical pdf
E $\mathbf{X}(t_i)$ are zero-mean random variables

- C only
- A, B, D
- D, E only
- B, C, E
- B, C, D, E
- A, B, C, D, E

No, the answer is incorrect.
Score: 0
Accepted Answers:
B, C, D, E

4) A process $\mathbf{X}(t)$ with autocorrelation function $R_{\mathbf{X}}(\tau) = e^{-\pi\tau^2}$ is passed through an LTI system. The magnitude of the frequency response $|H(f)|$ of the LTI system, such that the output process $\mathbf{Y}(t)$ is white, can be? *1 point*

- $e^{-\pi f^2}$
- $e^{\pi f^2}$
- $e^{-\frac{\pi f^2}{2}}$
- $e^{\frac{\pi f^2}{2}}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$e^{\frac{\pi f^2}{2}}$

GROUP-B

There are 4 questions in this section. Each question contains two marks.

5) Let random process $\mathbf{X}(t)$, with autocorrelation function as $R_{\mathbf{X}}(\tau) = e^{-2|\tau|}$ and mean $E[\mathbf{X}(t)] = 2.5$ is passed through an LTI filter with impulse response *2 points*

$$h(t) = \begin{cases} 2e^t, & 0 < t < \log_e 1.5 \\ t, & 1 < t < \sqrt{2} \\ 0, & \text{otherwise} \end{cases}$$

to produce $\mathbf{Y}(t)$ process at the output. What is $E[\mathbf{Y}(t)]$ at $t = 0.5\ s$?

- 0.25
- 1.5
- 3.75
- 5.0

No, the answer is incorrect.
Score: 0
Accepted Answers:
3.75

6) Let $\mathbf{X}(t)$ be a zero-mean wide sense stationary random process with its power spectral density given as $S_{\mathbf{X}}(f) = \text{rect}\left(\frac{f}{2}\right)$, is passed through an LTI system with transfer function $h(t) = \delta(t)$ and produces $\mathbf{Y}(t)$ process at the output. What is the value of cross-correlation function $R_{\mathbf{Y,X}}(0,2)$? [Hint: $R_{\mathbf{Y,X}}(\tau) = R_{\mathbf{X,Y}}(\tau)$ for a WSS process] *2 points*

- sinc(2)
- 2sinc(2)
- sinc(4)
- 2sinc(4)

No, the answer is incorrect.
Score: 0
Accepted Answers:
2sinc(4)

7) Let $\mathbf{X}(t)$ and $\mathbf{Y}(t)$ be two random processes described as follows: *2 points*

$\mathbf{X}(t) = A\cos(\omega_o t + \mathbf{\Phi})$, $\mathbf{\Phi}$ is a uniform random variable lying between $[0, 2\pi]$, A and ω_o are constants. What is its time average mean (temporal mean) and time autocorrelation (temporal autocorrelation) of $\mathbf{X}(t)$? [For temporal analysis, consider a sample function of process $\mathbf{X}(t)$ with $\mathbf{\Phi} = \theta_0$ constant value and integrating as $\lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} < \text{Expression} > dt$]

- $A, \frac{A^2}{2}\sin\omega_o\tau$
- $A, \frac{A}{2}\sin\omega_o\tau$
- $0, \frac{A^2}{2}\cos\omega_o\tau$
- $0, \frac{A}{2}\cos\omega_o\tau$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$0, \frac{A^2}{2}\cos\omega_o\tau$

8) A covariance matrix for a 3×1 Gaussian random vector $\underline{\mathbf{X}}$ is given as: *2 points*

$K_{\underline{\mathbf{X}}} = \begin{bmatrix} a & b & -c \\ b & m & x \\ -c & x & n \end{bmatrix}$ where variables a, b, c, m, n, x are positive real values and $\det|K_{\underline{\mathbf{X}}}| = 0$. Therefore, a linear transformation is performed on covariance matrix $K_{\underline{\mathbf{X}}}$ to attain a new covariance matrix of lower rank:

$K_{\underline{\mathbf{Y}}} = \begin{bmatrix} q & 0 \\ 0 & p \end{bmatrix}$ where variables q, p are positive real values and new random vector thus obtained is 2×1 Gaussian random vector $\underline{\mathbf{Y}}$. Then, which of the following is true?

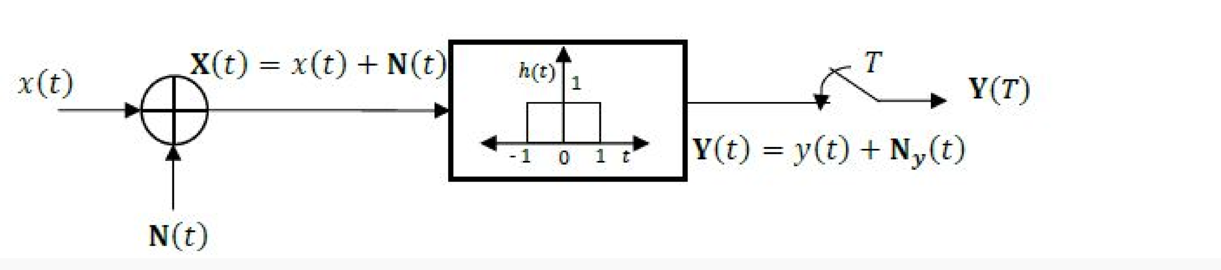
A At least one of the three random variables of vector $\underline{\mathbf{X}}$ is linearly dependent on other random variables of vector $\underline{\mathbf{X}}$
B It is not possible to represent a joint pdf of $\underline{\mathbf{X}}$ in 3D space without using impulses.
C Two random variables of vector $\underline{\mathbf{Y}}$ are statistically independent
D Two random variables of vector $\underline{\mathbf{Y}}$ are uncorrelated

- A, B, C, D
- A, B, D
- A, B
- A, D

No, the answer is incorrect.
Score: 0
Accepted Answers:
A, B, C, D

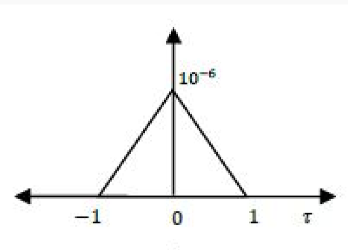
GROUP-C

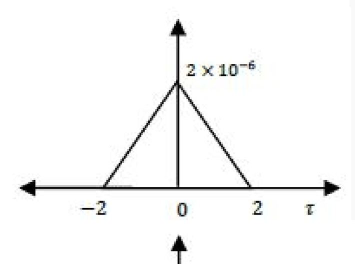
Let $x(t) = 2, 0 < t < 1$, be the information signal passed through an LTI filter having transfer function $h(t) = \text{rect}\left(\frac{t}{2}\right)$. Let $\mathbf{N}(t)$, a white Gaussian noise process having double-sided power spectral density of 10^{-6} Watts/Hz is added to the signal as shown in the diagram.

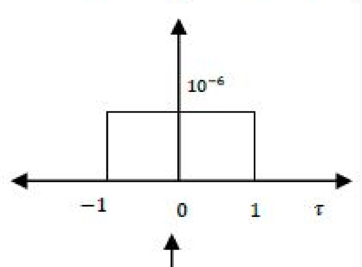


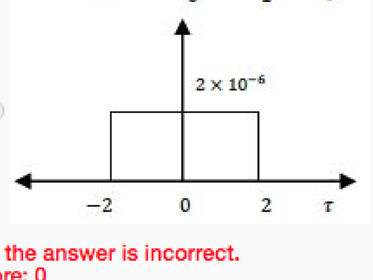
Answer the following 2 questions based on the above statement.

9) Which of the following represents a valid auto-correlation function $R_{\mathbf{N}_y}(\tau)$? *1 point*

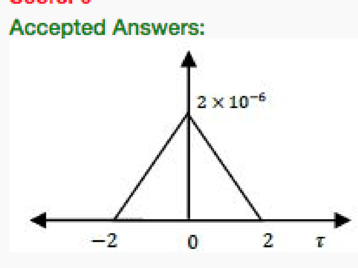
- 

- 

- 

- 

No, the answer is incorrect.
Score: 0
Accepted Answers:



10) The output of the filter is sampled at T. Find power of $\mathbf{Y}(T)$ i.e. $E[\mathbf{Y}^2(T)]$ at $T = 1$? *2 points*

$\left[\text{Hint: } y(t) = x(t) \otimes h(t);\ E[\mathbf{Y}^2(t)] = E\left[\left(y(t) + \mathbf{N}_y(t)\right)^2\right]\right]$

- $1+10^{-6}$
- $2+10^{-6}$
- $2(1.5+10^{-6})$
- $2(2+10^{-6})$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$2(2+10^{-6})$