Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis

Solutions for Week 6 Assignment

April 8, 2017

1. How does Translation affect the Fourier Transform of a function?

   (a) Changes only in the phase

   (b) Changes only in the magnitude

   (c) Changes in both phase and magnitude

   (d) Fourier transform remains unaffected

   Correct Answer : (a)

2. Choose the correct answer(s) for the Fourier Transform of Haar scaling function $\Phi(t)$ **(Multiple answers can be correct)**

   (a) $\frac{1+e^{j\Omega}}{j\Omega}$

   (b) $\frac{1-e^{-j\Omega}}{j\Omega}$

   (c) $e^{-j\Omega/2}\frac{Sin(\Omega/2)}{\Omega/2}$

   (d) $e^{-j\Omega/2}\frac{Cos(\Omega/2)}{\Omega/2}$

   Correct Answer : (b), (c)
   Hint: $\int_0^1 1.e^{-j\Omega t}dt$ **which on simplification gives options b,c.**

3. Which of the following affects the magnitude of the Fourier Transform of a function?

(a) Translation

(b) Dialation

(c) Both (a) and (b)

(d) None of the above

Correct Answer : (b)
Hint: Translation only affects the phase

4. Choose one or more correct statements from the following options

   (a) A function and its Fourier transform can both be compactly supported simultaneously

   (b) A function and its Fourier transform can not both be compactly supported simultaneously

   (c) We can Compactly support simultaneously in both time and frequency in some cases

   (d) None of the above options

   Correct Answer : (b)
   Hint: The uncertainty principle

5. Which of the following option(s) is/are equivalent to $< X(t), \Phi(t - \tau) >$ where X(t) is some function and $\Phi(t)$ is any scaling function. **(Multiple answers can be correct)**

   (a) $\int_{-\infty}^{\infty} X(t)\bar{\Phi}(t-\tau)d\tau$

   (b) $1/2\pi \int_{-\infty}^{\infty} \hat{\Phi}(\Omega)\hat{X}(\Omega)d\Omega$

   (c) $1/2\pi \int_{-\infty}^{\infty} \hat{X}(\Omega)\bar{\hat{\Phi}}(\Omega)e^{j\Omega\tau}d\Omega$

   (d) Inverse Fourier transform of $\hat{\Phi}(\Omega)\hat{X}(\Omega)$

   Correct Answer : (a), (c), (d)
   Hint: (a) is by definition of inner product, (c) is using parsevals theorem and fourier transform properties ,(d) is by definition of inverse fourier transform equivalent to (c)

6. Choose the correct Normalising factor for $\Phi(2^m t - n)$

(a) 2^m

(b) $2^{m/2}$

(c) 2^{-m}

(d) $2^{-m/2}$

Correct Answer : (b)
Hint: Take the inner product with itself and then square root

7. Choose the correct answer(s) for the Fourier Transform of Haar wavelet function $\Psi(t)$ **(Multiple answers can be correct)**

   (a) $1/2(1 - e^{-j\Omega})\hat{\Phi}(\Omega/2)$

   (b) $1/2(1 - e^{-j\Omega/2})\hat{\Phi}(\Omega/2)$

   (c) $2je^{-j\Omega/2}\frac{sin^2(\Omega/4)}{\Omega/4}$

   (d) $2je^{-j\Omega/2}\frac{sin^2(\Omega/2)}{\Omega/2}$

   Correct Answer : (b), (c)
   Hint: $\Psi(t) = \Phi(2t) - \Phi(2t-1)$ and then use fourier transform properties.

8. Choose valid option(s) from the following for the magnitude of Fourier Transform of Haar scaling function $\Phi(t)$. **(Multiple answers can be correct)**

   (a) $\hat{\Phi}(0) = 1$

   (b) $\hat{\Phi}(0) = 0$

   (c) $\hat{\Phi}(2m\pi) = 0, m \in \mathbb{Z} - (0)$

   (d) $\hat{\Phi}(m\pi) = 0, m \in \mathbb{Z} - (0)$

   Correct Answer : (a), (c)
   Hint: Φ has low pass character and also Sinc magnitude response

9. Choose valid option(s) from the following for the magnitude of Fourier Transform of Haar wavelet function $\Psi(t)$. **(Multiple answers can be correct)**

(a) Between 2π and 4π, magnitude response decreases continuously
(b) Zero at all even multiple of π
(c) Zero at $4m\pi$ where m $\in Z$
(d) All of the above statements are correct

Correct Answer : (a), (c)
Hint: Ψ has a magnitude response of the form $sin() * sinc()$

10. With increase in m, choose correct options associated with $\Phi(2^m t - n)$, where m,n $\in \mathbb{Z}$. **(Multiple answers can be correct)**

    (a) Emphasize larger and larger bands around 0 frequency
    (b) Emphasize smaller and smaller bands around 0 frequency
    (c) Narrowing in time
    (d) Broadening in time

    Correct Answer : (a), (c)
    Hint : Properties of dialates of Φ

11. As we go from W_0 to W_1 to W_2 and so on, what are the changes that take place in Fourier transform of wavelet function Ψ? **(Multiple answers can be correct)**

    (a) Mainlobe and sidelobes both expands
    (b) Mainlobe expands but sidelobes shrink
    (c) Mainlobe and sidelobe both shrinks
    (d) Center frequency (Point of maxima) doubles

    Correct Answer : (a), (d)
    Hint : Properties of dialates of Φ

12. What does going up the ladder correspond to in terms of time and frequency?

    (a) Expanding in both frequency and time
    (b) Expanding in frequency and contracting in time
    (c) Contracting in frequency and expanding in time

(d) Contracting in both frequency and time

Correct Answer : (b)

13. A Gaussian chirp $f(t) = exp(-(a - jb)t^2$ has a fourier transform

(a) $\hat{f}(\omega) = \sqrt{\frac{\pi}{(a-jb)}} exp(\frac{-(a+jb)\omega^2}{4(a^2+b^2})$

(b) $\hat{f}(\omega) = \sqrt{\frac{\pi}{a^2+b^2}} exp(\frac{-(a-jb)\omega^2}{4(a^2+b^2})$

(c) $\hat{f}(\omega) = \sqrt{\frac{\pi}{ab}} exp(\frac{-(a-jb)\omega^2}{4(a^2+b^2})$

(d) None of the above

Correct Answer : (a)
Note that for Gaussian $f(t) = e^{-t^2}$ **we have** $\hat{f}(\omega) = \sqrt{\pi} e^{-\omega^2/4}$
Now apply the scaling property of fourier transform to get the required result

14. If for the function f(t), the fourier transform is $\hat{(f)}(\omega)$ and if it obeys: $\int_{-\infty}^{\infty} |\hat{f}(\omega)|(1 + |\omega^p|)d\omega < \infty$, where p is a positive integer, then :

(a) $\forall k \leqslant p, \frac{d^k}{dt^k} f(t)$ is bounded

(b) $\forall k \leqslant p, \frac{d^k}{dt^k} \hat{f}(\omega)$ is bounded

(c) $\forall k \in Z, \frac{d^k}{dt^k} \hat{f}(\omega)$ is bounded

(d) None of the above

Correct answer : (a)
$|f(t)| \leqslant 1/2\pi \int_{-\infty}^{\infty} |\hat{f}(\omega)| d\omega$
Also we know that :
$\forall k \leqslant p, \frac{d^k}{dt^k} f(t) < -- > (i\omega)^k \hat{f}(\omega$
$==> |\frac{d^k}{dt^k} f(t)| \leqslant \int_{-\infty}^{\infty} |\omega^k||\hat{f}(\omega)| d\omega$

15. If $f(t) \in L^2(R)$ and if we define a new function $f_s(t) = 1/\sqrt{s} f(t/s)$ and if ——.—— denotes L^2 - norm, then:

(a) $||f(t)|| = \sqrt{s}||f_s(t)||$

(b) $||f(t)|| = 1/\sqrt{s}||f_s(t)||$

(c) $||f(t)|| = s||f_s(t)||$

(d) $||f(t)|| = ||f_s(t)||$

Correct answer : (d)
$||f_s(t)|| = (\int_{-\infty}^{\infty} 1/s f^2(t/s)dt)^{1/2}$
$t/s = \tau$ **gives** $sd\tau = dt$
$=(\int_{-\infty}^{\infty} f^2(\tau)d\tau)^{1/2} = ||f(t)||$

16. What happens when we go down the ladder, choose the correct statements **(Multiple answers can be correct)**

   (a) Φ emphasises smaller and smaller bands around 0

   (b) Φ emphasises larger and larger bands around 0

   (c) Width and Center frequency decreases geometrically of Ψ mainlobe

   (d) Width and Center frequency increases geometrically of Ψ mainlobe

   Correct Answer : (a), (c)

17. For f $\in L^2(R)$, we have a linear time-frequency transform
$T(f(\tau)) = \int_{-\infty}^{\infty} f(t)\phi_\tau^*(t)dt$
where ϕ_τ is called a time-frequency atom and $\phi_\tau \in L^2(R)$
and $||\phi_\tau|| = 1$, then

   (a) $T(f(\gamma) = 1/2\pi \int_{-\infty}^{\infty} \hat{f}(\omega) * \hat{\phi_\gamma}(\omega)d\omega$

   (b) $T(f(\gamma) = 1/2\pi \int_{-\infty}^{\infty} \hat{f}(\omega)\hat{\phi_\gamma}(\omega)d\omega$

   (c) $T(f(\gamma) = 1/2\pi \int_{-\infty}^{\infty} \hat{f}(\omega)\hat{\phi_\gamma^*}(\omega)d\omega$

   (d) None of the above

   Correct answer : (c)
$T(f(\gamma) =< f, \phi_\tau >$
using Parseval's Theorem $=> 1/2\pi < \hat{f}, \hat{\phi_\tau} >= 1/2\pi \int_{-\infty}^{\infty} \hat{f}(\omega)\hat{\phi_\gamma^*}(\omega)d\omega$