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NPTEL

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Courses » An Introduction to Probability in Computing

Announcements **Course** Ask a Question Progress Mentor

Unit 6 - Week 4

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Ineraction Session

Assignment 4

The due date for submitting this assignment has passed. **Due on 2018-03-07, 23:59 IST.**

Submitted assignment

1) Let X be a random variable having pmf. *1 point*

X	$p(x)$
0	0.4
1	0.2
3	0.4

What is the moment generating function of X.

- $0.4\ e^{3t}$
- $0.2\ e^{t}$
- $0.2\ e^{t} + 0.4\ e^{3t}$
- None of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:
None of the above

2) Let X and Y be independent random variables, then, $M_{X+Y}(t) =$ *1 point*

- $M_X(t).\ M_Y(t)$
- $M_X(t)/M_Y(t)$
- $M_X(t) + M_Y(t)$
- None of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:
$M_X(t).\ M_Y(t)$

3) Let X be a non-negative random variables with $E[X] = 5.4$ and $Var[X] = 0.2$, then, using *1 point* the second moment method (essentially, the Chebyshev's inequality) studied in the last segment of week

4, we can conclude that

- $Pr(X = 0) \leq 0.68$
- $Pr(X = 0) \leq 0.0068$
- $Pr(X = 0) \geq 0.68$
- $Pr(X = 0) \geq 0.0068$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$Pr(X = 0) \leq 0.0068$

4) While considering tail bounds on sum of random variables, Markov's inequality and Chebyshev's inequality do not require independence of random variables, but Chernoff's bound requires that the random variables be independent. ***1 point***

- True
- False

No, the answer is incorrect.
Score: 0

Accepted Answers:
True

5) Consider a biased coin with probability $p = 1/3$ of landing heads. Suppose the coin is flipped n times, and let X_i be a random variable denoting the i^{th} flip, where $X_i = 1$ means heads, and $X_i = 0$ means tails. Use the Chernoff bound ***1 point***

$$Pr[X \geq (1 + \delta)\mu] \leq e^{-\mu\delta^2/3}$$

for $0 < \delta \leq 1$ and $\mu = E[X]$ to determine the smallest value for n so that the probability that at least half the coin flips come out heads is less that 0.001.

- 198
- 249
- 307
- 349

No, the answer is incorrect.
Score: 0

Accepted Answers:
249

The purpose of this question is to show the limitations of Chernoff bound. In a match of football, Indian team made 15 shots at the goal. The team has a conversion probability of $p = 0.1$ (i.e. the probability that a shot at the goal actually becomes a goal is 0.1), but we managed to score only 1 goal.

6) Use Chernoff bound (Hint: for $0 < \delta < 1$) to bound $q = Pr[\text{\# of goals after 15 shots} \leq 1]$ ***1 point***

- $q \geq 1.823$
- $q \leq 1.823$
- $q \geq 0.909$
- $q \leq 0.909$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$q \leq 0.909$

7) Use standard probability to find $q = Pr[\text{\# of goals after 15 shots} \leq 1]$ *1 point*

- $q = 0.549$
- $q = 0.623$
- $q = 0.823$
- $q = 1.823$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$q = 0.549$

Consider the random graph $G_{n,p}$ on n vertices, where the probability of an edge between any two vertices in the graph is p (no multiple edges or self loops). Now, consider the random graph $G_{n,1/2}$,

8) What is the expected number of neighbours of any node v? *1 point*

- n
- $n/4$
- $\frac{n-1}{2} \approx n/2$
- $\sqrt{n}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\frac{n-1}{2} \approx n/2$

9) In this question, we want to show that almost all random graphs drawn from $G_{n,0.5}$ have minimum degree $d = (\frac{n}{2} - \sqrt{n} \ln n)$ What is the smallest upper bound one can derive for $p = Pr[|N(v)| \leq n/2 - \sqrt{n} \ln n]$using the appropriate Chernoff bounds? (Here, $N(v)$ is the neighbourhood of vertex v.) *1 point*

Select all applicable answers:

- $p \leq 1/n$
- $p \leq 1/n^{\ln n}$
- $p \leq e^{\ln^2 n}$
- none of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:
$p \leq 1/n^{\ln n}$
$p \leq e^{\ln^2 n}$

10)We wish to show that almost all nodes in $G_{n,0.5}$ have degree concentrated in the range $((\frac{n}{2} - \sqrt{3n/2} \ln n, \frac{n}{2} + \sqrt{3n/2} \ln n)$ What is the smallest upper bound one can derive for *1 point*

$p = Pr[|d - n/2| \leq \sqrt{3n/2}\ln n]$? (You must use the Chernoff bound for both tails which states that for $0 < \delta \leq 1$, $Pr[|X - \mu| \geq \delta\mu] \leq 2e^{-\mu\delta^2/3}$)

Select all applicable answers:

- [ ] $p \leq 2/n$
- [ ] $p \leq 2/n^{\ln n}$
- [ ] $p \leq 2e^{\ln^2 n}$
- [ ] none of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:
$p \leq 2/n^{\ln n}$
$p \leq 2e^{\ln^2 n}$

11) Consider random graphs in $G_{n,0.5}$. One of the questions the instructor asked at the end of the segment on random graphs was to show that the diameter is at most 2 with high probability. What can you say about the probability that the diameter is 1 (i.e., the graph is a clique)? Put another way, what can you say about $p =$ Pr(all the pairs are connected)? ***1 point***

Select all applicable answers:

- [ ] $p \to 0, \text{ as } n \to \infty$
- [ ] $1/4$
- [ ] $p = 2^{-\binom{n}{2}}$
- [ ] $p = 2^{-n}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$p \to 0, \text{ as } n \to \infty$
$p = 2^{-\binom{n}{2}}$

Given a population of N people, we want to estimate the fraction of people who would vote for $\underline{mickeymouse}$ in the coming election. We have decided to conduct a survey of n people chosen uniformly at random (UAR). Let's approximate the true fraction p by the fraction of people in the sample($\tilde{p}$) who support $\underline{mickeymouse}$.

12) Use Chernoff bound to find a bound on n, such that, $Pr(p \notin [\tilde{p} - \epsilon, \tilde{p} + \epsilon]) \leq \delta$ ***1 point***

- ( ) $n \geq \frac{3p}{\epsilon^2} \ln \frac{2}{\delta}$
- ( ) $n = O(\frac{1}{\epsilon^2} \ln \frac{2}{\delta})$
- ( ) All of the above
- ( ) None of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:
All of the above

13) What is the relation of n to N ***1 point***

$n \in \Theta(n)$

$n \in \Theta(\log n)$

$n \in O\left(\frac{\ln N}{\epsilon^2 \delta}\right)$

n is independent of N.

No, the answer is incorrect.
Score: 0

Accepted Answers:
n is independent of N.

(Chernoff vs Chebyshev's) We toss a fair coin n times. Let S_n be the number of heads we have observed.

14) Use Chebyshev's inequality to find $q = Pr\left[\left|\frac{S_n}{n} - \frac{1}{2}\right| \geq \frac{1}{4}\right]$ ***1 point***

$q \geq 4/n$

$q \leq 4/n$

$q \geq 16/n$

$q \leq 16/n$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$q \leq 4/n$

15) Which of the following is the best bound you can guarantee for $q = Pr\left[\left|\frac{S_n}{n} - \frac{1}{2}\right| \geq \frac{1}{4}\right]$ ***1 point***
using an appropriate Chernoff bound. (Hint: use the Chernoff bound defined for both tails and for $0 < \delta \leq 1$).

$q \geq 24/n$

$q \leq 24/n$

$q \geq 2e^{-n/24}$

$q \leq 2e^{-n/24}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$q \leq 2e^{-n/24}$

Consider a solution for permutation routing on an n-dimensional hypercube in which packets are deterministically routed to their destinations via bit-fixing.

16) Which of these statements are correct for the case where $n = 5$? ***1 point***
1.Suppose the packet from $(0, 0, 0, 0, 0)$ is destined for $(1, 1, 1, 1, 1)$. That packet takes ≥ 5 steps
2.Routing from $(0, 0, 0, 0, 0)$ to $(1, 1, 1, 1, 1)$ takes fewer than 5 steps
3.Suppose the packet from $(0, 0, 0, 0, 0)$ is destined for some arbitrary $(b_1, b_2, b_3, b_4, b_5)$. That packet always takes at least 5 steps regardless of the b_i values.
4.Routing from any $(a_1, a_2, a_3, a_4, a_5)$ to $(b_1, b_2, b_3, b_4, b_5)$ never exceeds 5 steps

1

1 and 3

- 2
- 2 and 4

No, the answer is incorrect.
Score: 0

Accepted Answers:
1

17) The number of rounds required for the routing to take place is **1 point**

- $O(\log n)$
- $\Omega(n)$
- $\Theta(\sqrt{n})$
- None of these.

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\Omega(n)$

Consider the Two-Phase routing algorithm for permutation routing on a hypercube.

18) Consider the bit-fixing algorithm for permutation routing. Which among the following is the smallest upper bound for the average number of packets that traverse the edges of the hypercube. **1 point**

- $O(1)$
- $O(\log n)$
- $O(\sqrt{n})$
- $O(n)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$O(1)$

19) Consider Phase-II, assuming that all packets complete their Phase-I. How is Phase-II accounted for? **1 point**

- Phase-II is exactly same as Phase-I
- Phase-II is exactly same as Phase-I running backward in the sense that each packet starts at a random
origin that is distinct from the origin of all other packets and ends at a destination that is also distinct from the destinations of all other packets.
- Phase-II is exactly same as Phase-I running backward in the sense that each packet starts at random origin (allowing the possibility that multiple packets can have the same origin) and end at a given destination that is distinct from the destinations of all other packets..
- None of these.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Phase-II is exactly same as Phase-I running backward in the sense that each packet starts at random origin (allowing the possibility that multiple packets can have the same origin) and end at a given destination that is distinct from the destinations of all other packets..

20) **1 point**

Consider a random variable X taking non-negative values. Then, $E(X) \rightarrow \infty \implies Pr(X = 0) < \epsilon$

- True
- False

No, the answer is incorrect.
Score: 0

Accepted Answers:
False

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