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Course

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#### Courses » An Introduction to Probability in Computing

Unit 5 - Week 3

# Course outline

Assignment 3

Submitted assignment

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Week 4

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Ineraction Session 1) Consider two unbiased coins that are flipped independently. Let  $X \in \{0, 1\}$  and  $Y \in \{0, 1\}$  **1** point be two random variables denoting their outcomes with 1 indicating heads and 0 indicating tails. Suppose we limit the sample space of the experiment to  $S = \{01, 10, 11\}$  instead of the actual  $S = \{00, 01, 10, 11\}$ . What is the co-variance of the two random variables?

The due date for submitting this assignment has passed. Due on 2018-02-28, 23:59 IST.

No, the answer is incorrect. Score: 0

Accepted Answers:

 $\frac{-1}{9}$ 

 $\frac{1}{2}$ 

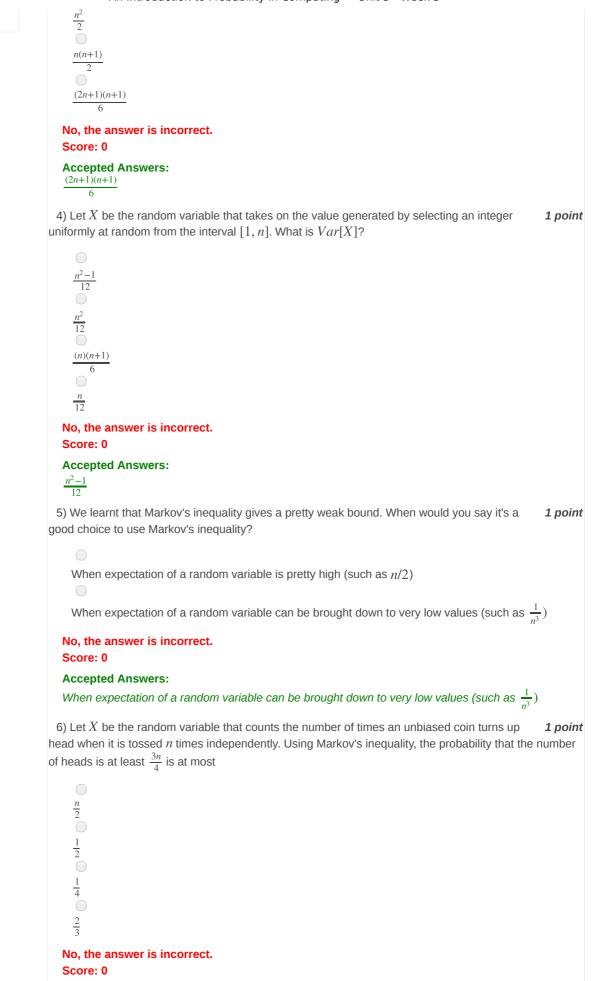
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 $\frac{-1}{9}$ 

2) Let *X* be the random variable that takes on the value generated by selecting an integer uniformly at random from the interval [1, n]. What is E[X]?

0 1		
0		
$\frac{n}{2}$		
$\frac{n+1}{2}$		
$\frac{1}{n}$		
No, the answer is incorrect. Score: 0		
Accepted Answers: $\frac{n+1}{2}$		
3) Let $X$ be the random variable that takes on the value generated by selecting an integer uniformly at random from the interval $[1, n]$ . What is the second moment of $X$ ?	1 point	
$n^2$		



1 point

 $\frac{2}{3}$ 7)

**Accepted Answers:** 

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For a 0 - 1 random variable X, is  $E[X^2] = E[X]$ ? What about  $E[X^n]$  for some large integer *n*? Choose which one of the following statements is correct.

 $E[X^{2}] = E[X] \text{ and } E[X^{n}] = E[X]$   $E[X^{2}] \neq E[X] \text{ but } E[X^{n}] = E[X]$   $E[X^{2}] = E[X] \text{ but } E[X^{n}] \neq E[X]$   $E[X^{2}] \neq E[X] \text{ and } E[X^{n}] \neq E[X]$ 

No, the answer is incorrect. Score: 0

## Accepted Answers: $E[X^2] = E[X]$ and $E[X^n] = E[X]$

8) Let X be the random variable that counts the number of times heads turns up when an unbiased coin is tossed n times. Recall that the Chebyshev's inequality states:

 $Pr(|X - E[X]| \ge a) \le \frac{Var(X)}{a^2}$  Using Chebyshev's inequality, the probability the number of heads is at least  $\frac{3n}{4}$  is at most

No, the answer is incorrect. Score: 0

Accepted Answers:

 $\frac{4}{n}$ 

 $\frac{n}{2}$ 

 $\frac{4}{n}$ 

 $\frac{1}{n}$ 

 $\frac{2}{3}$ 

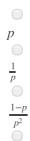
9) Consider a random variable X. If for a constant c, E[cX] = cE[X], what is Var(cX)? **1** point

Var(cX) = cVar(X)  $Var(cX) = \frac{1}{c} Var(X)$   $Var(cX) = c^{2}Var(X)$  Var(cX) = 0

No, the answer is incorrect. Score: 0

Accepted Answers:  $Var(cX) = c^2 Var(X)$ 

10)What is the variance of a geometric random variable that has a probability *p* of success? **1** point





No, the answer is incorrect. Score: 0

Accepted Answers: 1-p

 $p^2$ 11) Let's consider an experiment in which a fair coin is tossed n times to obtain n random bits. 1 point Consider  $m = \binom{n}{2}$  pairs of these bits in some order. Let  $Y_i$  be the exclusive-or of the  $i^{th}$  pair of bits. Let  $Y = \sum_{i=1}^{m} Y_i$  be the number of  $Y_i$  that equal to 1. What is the probability that any given  $Y_i$  is 0?  $\frac{1}{4}$  $\frac{1}{m}$  $\frac{1}{n}$  $\frac{1}{2}$ No, the answer is incorrect. Score: 0 **Accepted Answers:**  $\frac{1}{2}$ 12) n the experiment described in 11, are any two given random variables  $Y_i$  and  $Y_i$  guaranteed **1** point to be always independent? No Yes No, the answer is incorrect. Score: 0 **Accepted Answers:** Yes 13) n the experiment described in 11, is  $E[Y_iY_j] = E[Y_i]E[Y_j]$ ? 1 point No Yes No, the answer is incorrect. Score: 0 **Accepted Answers:** Yes 14For a coin that comes up heads with probability p on each flip, let X denote the number of **1** point flips until the  $k^{th}$  head appears. What is the Var[X]?  $\frac{1}{k}$  $\frac{k(1-p)}{p^2}$  $\frac{1}{1-k}$ 

No, the answer is incorrect. Score: 0 Accepted Answers:

1-p



Consider the median finding algorithm for finding the median m of a set S of n items from an ordered universe on page~54 of the textbook ``Probability and Computing: Randomized Algorithms and Probabilistic Analysis" by Michael Mitzenmacher and Eli Upfal (First Edition) 1. The following **four** questions are based on your comprehension of the algorithm and its analysis. (Please note that this material is being used for educational purposes in this assignment.)

15) Additional material for question 15 to 18 CLICK HERE

Can the median finding algorithm output an element in S that is not the median?

yesNo

No, the answer is incorrect. Score: 0

Accepted Answers: No

16)Which step in the algorithm guarantees the outcome presented in the previous question? **1** point

	Step 6
$\bigcirc$	Step 7
$\bigcirc$	Step 8
$\bigcirc$	Step 5

No, the answer is incorrect. Score: 0

Accepted Answers: Step 6

17)Would any of the following conditions lead the algorithm to fail in finding the median? Choose**1** point all that apply.

```
When median is in C
When median is not in C
When the set is sorted
When median is n/2
None of the above
```

No, the answer is incorrect. Score: 0

Accepted Answers: When median is not in C

18)Consider the following description of events for the median finding algorithm from the **1** point textbook ``Probability and Computing: Randomized Algorithms and Probabilistic Analysis" by Michael Mitzenmacher and Eli Upfal (First Edition) 1, where *m* is the median:  $\mathcal{E}_1 : |C| > 4n^{3/4}$ 

 $\begin{aligned} \mathcal{E}_2 &: Y_1 = |\{r \in R | r \leq m\}| < 1/2n^{3/4} - \sqrt{n} \\ \mathcal{E}_3 &: Y_3 = |\{r \in R | r \geq m\}| < 1/2n^{3/4} - \sqrt{n} \\ \text{When will the algorithm fail? Choose all options that may apply.} \end{aligned}$ 

None of the events happen.

If  $\mathcal{E}_1$  happens.

1 point

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- If at least one of the three events occur.
- If all three events occur.

## No, the answer is incorrect. Score: 0

Accepted Answers: If  $\mathcal{E}_1$  happens. If at least one of the three events occur. If all three events occur.

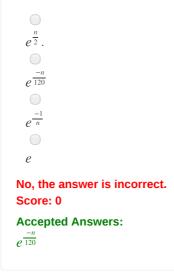
19)What is the moment generating function  $M_X(t)$  of a Bernoulli random variable X which has a **1** *point* probability p of success?

 $pe^{t}$   $1 + pe^{t+1}$   $1 + p(e^{t} - 1)$   $1 + pe^{t-1}$ No, the answer is incorrect.

Score: 0 Accepted Answers:  $1 + p(e^t - 1)$ 

20)Alice and Bob play chess often. Alice wins any given game they play with probability 0.6. **1** *point* They both decide to play *n* games in total. Use the following Chernoff bound to bound the probability that

Alice loses more than half the games they play. For  $0 \le \delta \le 1$ ,  $Pr(X \ge (1 + \delta)\mu) \le e^{\frac{-\mu\delta^2}{3}}$  Choose the answer that is most appropriate.



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