X

NPTEL

reviewer1@nptel.iitm.ac.in ▼

Courses » An Introduction to Probability in Computing

Announcements Course Ask a Question Progress Mentor

Unit 5 - Week 3

Assignment 3

The due date for submitting this assignment has passed. **Due on 2018-02-28, 23:59 IST.**

Submitted assignment

1) Consider two unbiased coins that are flipped independently. Let $X \in \{0, 1\}$ and $Y \in \{0, 1\}$ be two random variables denoting their outcomes with 1 indicating heads and 0 indicating tails. Suppose we limit the sample space of the experiment to $S = \{01, 10, 11\}$ instead of the actual $S = \{00, 01, 10, 11\}$. What is the co-variance of the two random variables? ***1 point***

- $\frac{1}{2}$
- $\frac{3}{4}$
- $\frac{-1}{3}$
- $\frac{-1}{9}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\frac{-1}{9}$

2) Let X be the random variable that takes on the value generated by selecting an integer uniformly at random from the interval $[1, n]$. What is $E[X]$? ***1 point***

- 1
- $\frac{n}{2}$
- $\frac{n+1}{2}$
- $\frac{1}{n}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\frac{n+1}{2}$

3) Let X be the random variable that takes on the value generated by selecting an integer uniformly at random from the interval $[1, n]$. What is the second moment of X? ***1 point***

- n^2
-

$\frac{n^2}{2}$

$\frac{n(n+1)}{2}$

$\frac{(2n+1)(n+1)}{6}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\frac{(2n+1)(n+1)}{6}$

4) Let X be the random variable that takes on the value generated by selecting an integer uniformly at random from the interval $[1, n]$. What is $Var[X]$? ***1 point***

$\frac{n^2-1}{12}$

$\frac{n^2}{12}$

$\frac{(n)(n+1)}{6}$

$\frac{n}{12}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\frac{n^2-1}{12}$

5) We learnt that Markov's inequality gives a pretty weak bound. When would you say it's a good choice to use Markov's inequality? ***1 point***

When expectation of a random variable is pretty high (such as $n/2$)

When expectation of a random variable can be brought down to very low values (such as $\frac{1}{n^3}$)

No, the answer is incorrect.
Score: 0

Accepted Answers:
When expectation of a random variable can be brought down to very low values (such as $\frac{1}{n^3}$)

6) Let X be the random variable that counts the number of times an unbiased coin turns up head when it is tossed n times independently. Using Markov's inequality, the probability that the number of heads is at least $\frac{3n}{4}$ is at most ***1 point***

$\frac{n}{2}$

$\frac{1}{2}$

$\frac{1}{4}$

$\frac{2}{3}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\frac{2}{3}$

7) ***1 point***

For a $0 - 1$ random variable X, is $E[X^2] = E[X]$? What about $E[X^n]$ for some large integer n? Choose which one of the following statements is correct.

- $E[X^2] = E[X]$ and $E[X^n] = E[X]$
- $E[X^2] \neq E[X]$ but $E[X^n] = E[X]$
- $E[X^2] = E[X]$ but $E[X^n] \neq E[X]$
- $E[X^2] \neq E[X]$ and $E[X^n] \neq E[X]$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$E[X^2] = E[X]$ and $E[X^n] = E[X]$

8) Let X be the random variable that counts the number of times heads turns up when an unbiased coin is tossed n times. Recall that the Chebyshev's inequality states: $Pr(|X - E[X]| \geq a) \leq \frac{Var(X)}{a^2}$ Using Chebyshev's inequality, the probability the number of heads is at least $\frac{3n}{4}$ is at most ***1 point***

- $\frac{n}{2}$
- $\frac{4}{n}$
- $\frac{1}{n}$
- $\frac{2}{3}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\frac{4}{n}$

9) Consider a random variable X. If for a constant c, $E[cX] = cE[X]$, what is $Var(cX)$? ***1 point***

- $Var(cX) = cVar(X)$
- $Var(cX) = \frac{1}{c} Var(X)$
- $Var(cX) = c^2 Var(X)$
- $Varc(cX) = 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$Var(cX) = c^2 Var(X)$

10)What is the variance of a geometric random variable that has a probability p of success? ***1 point***

- p
- $\frac{1}{p}$
- $\frac{1-p}{p^2}$

$\frac{1-p}{p}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\frac{1-p}{p^2}$

11) Let's consider an experiment in which a fair coin is tossed n times to obtain n random bits. Consider $m = \binom{n}{2}$ pairs of these bits in some order. Let Y_i be the exclusive-or of the i^{th} pair of bits. Let $Y = \sum_{i=1}^{m} Y_i$ be the number of Y_i that equal to 1. What is the probability that any given Y_i is 0? ***1 point***

- $\frac{1}{4}$
- $\frac{1}{m}$
- $\frac{1}{n}$
- $\frac{1}{2}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\frac{1}{2}$

12) In the experiment described in 11, are any two given random variables Y_i and Y_j guaranteed to be always independent? ***1 point***

- No
- Yes

No, the answer is incorrect.
Score: 0

Accepted Answers:
Yes

13) In the experiment described in 11, is $E[Y_iY_j] = E[Y_i]E[Y_j]$? ***1 point***

- No
- Yes

No, the answer is incorrect.
Score: 0

Accepted Answers:
Yes

14) For a coin that comes up heads with probability p on each flip, let X denote the number of flips until the k^{th} head appears. What is the $Var[X]$? ***1 point***

- $\frac{1}{k}$
- $\frac{k(1-p)}{p^2}$
- $\frac{1}{1-k}$
- $\frac{1-p}{p^2}$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$\frac{k(1-p)}{p^2}$

Consider the median finding algorithm for finding the median m of a set S of n items from an ordered universe on page~54 of the textbook ``Probability and Computing: Randomized Algorithms and Probabilistic Analysis" by Michael Mitzenmacher and Eli Upfal (First Edition) 1. The following **four** questions are based on your comprehension of the algorithm and its analysis. (Please note that this material is being used for educational purposes in this assignment.)

15) *1 point*

Additional material for question 15 to 18

CLICK HERE

Can the median finding algorithm output an element in S that is not the median?

- yes
- No

No, the answer is incorrect.
Score: 0

Accepted Answers:
No

16) Which step in the algorithm guarantees the outcome presented in the previous question? *1 point*

- Step 6
- Step 7
- Step 8
- Step 5

No, the answer is incorrect.
Score: 0

Accepted Answers:
Step 6

17) Would any of the following conditions lead the algorithm to fail in finding the median? Choose all that apply. *1 point*

- When median is in C
- When median is not in C
- When the set is sorted
- When median is $n/2$
- None of the above

No, the answer is incorrect.
Score: 0

Accepted Answers:
When median is not in C

18) Consider the following description of events for the median finding algorithm from the textbook ``Probability and Computing: Randomized Algorithms and Probabilistic Analysis" by Michael Mitzenmacher and Eli Upfal (First Edition) 1, where m is the median: $\mathcal{E}_1 : |C| > 4n^{3/4}$ *1 point*

$\mathcal{E}_2 : Y_1 = |\{r \in R | r \leq m\}| < 1/2n^{3/4} - \sqrt{n}$

$\mathcal{E}_3 : Y_3 = |\{r \in R | r \geq m\}| < 1/2n^{3/4} - \sqrt{n}$ When will the algorithm fail? Choose all options that may apply.

- None of the events happen.
- If $\mathcal{E}_1$ happens.

- [ ] If at least one of the three events occur.
- [ ] If all three events occur.

No, the answer is incorrect.
Score: 0

Accepted Answers:
If $\mathcal{E}_1$ happens.
If at least one of the three events occur.
If all three events occur.

19) What is the moment generating function $M_X(t)$ of a Bernoulli random variable X which has a probability p of success? ***1 point***

- pe^t
- $1 + pe^{t+1}$
- $1 + p(e^t - 1)$
- $1 + pe^{t-1}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$1 + p(e^t - 1)$

20) Alice and Bob play chess often. Alice wins any given game they play with probability $0.6.$ They both decide to play n games in total. Use the following Chernoff bound to bound the probability that Alice loses more than half the games they play. For $0 \leq \delta \leq 1, Pr(X \geq (1+\delta)\mu) \leq e^{\frac{-\mu\delta^2}{3}}$ Choose the answer that is most appropriate. ***1 point***

- $e^{\frac{n}{2}}.$
- $e^{\frac{-n}{120}}$
- $e^{\frac{-1}{n}}$
- e

No, the answer is incorrect.
Score: 0

Accepted Answers:
$e^{\frac{-n}{120}}$

Previous Page

End

A project of

NPTEL National Programme on Technology Enhanced Learning

In association with

NASSCOM®

Funded by

Government of India
Ministry of Human Resource Development

Powered by

Google™