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Courses » An Introduction to Probability in Computing

Announcements **Course** Ask a Question Progress Mentor

Unit 4 - Week 2

Week 2 - Assignment 1

The due date for submitting this assignment has passed. **Due on 2018-02-21, 23:59 IST.**

Submitted assignment

For questions 14-17, we refer to external material from a textbook, which is attached here

1) Consider the event that two dice are thrown and the numbers on their top faces are recorded. Let X be a random variable representing the sum of the recorded numbers. What is Pr(X = 5) equal to? ***1 point***

- 1/5
- 1/7
- 1/9
- 1/11

No, the answer is incorrect.
Score: 0

Accepted Answers:
1/9

2) Consider a special weighted die that when thrown will result in each of its faces appearing on top with probability distribution of (1/4, 1/6, 1/6, 1/5, 1/8, 11/120) assigned to the faces having numbers (1, 8, 3, 2, 5, 7). Let X be a random variable representing the value of the face appearing on top after one throw of this die. What is the expected value of X? ***1 point***

- 2.66
- 3
- 3.75
- 4.33

No, the answer is incorrect.
Score: 0

Accepted Answers:
3.75

3) We are given an unbiased die with faces containing the numbers 1, 2, 3, 4, 5, 6 and an unbiased coin with faces containing the numbers 2, 4. Let X_1 be a random variable denoting the value of the number on top of the die after throwing it once. Let X_2 be a random variable denoting the value of the number on top of the coin after throwing it once. Let $X = X_1 + X_2$. What is the expected value of X? ***1 point***

- 3.25
- 3.875
- 6.5

Week 2 Feedback

Solutions to Week 2 assignment

Week 3

Week 4

Download

Ineraction Session

- 9

No, the answer is incorrect.
Score: 0

Accepted Answers:
6.5

4) Consider the event that an unbiased die with six faces containing the numbers 1, 2, 3, 4, 5, 6 is thrown and the number on the top face is taken. Let X be a random variable that takes value 1 if the number is 1, takes value 2 if the number is 2, and takes value 0 otherwise. Select all of the options below which are true. ***1 point***

- The expected value of X is 3.5.
- The expected value of X is 0.5.
- X is a Bernoulli random variable.
- X is a Binomial random variable.
- X is a geometric random variable.

No, the answer is incorrect.
Score: 0

Accepted Answers:
The expected value of X is 0.5.

5) Consider that we are flipping 5 weighted coins, where for each coin the probability of coming up heads is 1/4 and that of coming up tails is 3/4. What is the probability that after flipping the 5 coins, we'll end up with exactly 2 heads? ***1 point***

- 9/1024
- 27/1024
- 90/1024
- 270/1024

No, the answer is incorrect.
Score: 0

Accepted Answers:
270/1024

6) Consider two unbiased dice with numbers 1 to 6 on their faces. Suppose we throw the two dice. Let X_1 and X_2 be the random variables denoting which numbers appear on the top faces of the first and second die respectively. ***1 point***
Let $X = X_1 + X_2$. What is the expected value of X given that we know that $X_2 = 3.$.

- 5.5
- 6.5
- 7.5
- 8.5

No, the answer is incorrect.
Score: 0

Accepted Answers:
6.5

7) Consider a weighted coin that when thrown will land heads up with probability 1/4 and tails up with probability 3/4. Suppose we now toss this coin until we get the first heads. What is the probability that it will take exactly 5 tosses to get the first heads? ***1 point***

- 81/256
- 81/1024
- 243/1024
- 243/4096

No, the answer is incorrect.
Score: 0

Accepted Answers:

81/1024

8) For the above problem what is the expected number of tosses required until we get the first heads. ***1 point***

- 4/3
- 16/9
- 4
- 16

No, the answer is incorrect.
Score: 0

Accepted Answers:
4

9) Consider a geometric random variable X with parameter p. Then what is the relationship between Pr(X = n + k + 2 | X > k + 1) and Pr(X = n) for n > 0? ***1 point***

- $Pr(X = n + k + 2 \mid X > k + 1) = Pr(X = n)/(1 - p)$
- $Pr(X = n + k + 2 \mid X > k + 1) = Pr(X = n)$
- $Pr(X = n + k + 2 \mid X > k + 1) = (1 - p)Pr(X = n)$
- $Pr(X = n + k + 2 \mid X > k + 1) = (1 - p)^2 Pr(X = n)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$Pr(X = n + k + 2 \mid X > k + 1) = (1 - p)Pr(X = n)$

For questions 10-13, consider the following game. There exists a bag containing the same number of blue, green, yellow, orange, and red balls. However, you are not told how many balls are in the bag in total. In each round of the game, you are required to pick a ball from the bag (without looking in the bag) and record its color. You subsequently put the ball back in the bag and the bag is then shaken up. Your goal is to play the game for as many rounds as possible until you've picked a ball of each color.

10)Assume that you initially picked a blue ball in round one and a green ball in round two. How many rounds on expectation will you have to play until you pick a new color ball? ***1 point***

- 5
- 5/2
- 5/3
- 5/4
- 1

No, the answer is incorrect.
Score: 0

Accepted Answers:
5/3

11)Assume that you initially picked a blue ball in round one. However, for rounds two to seven, you kept picking a blue ball. Finally in round eight, you picked a yellow ball. How many rounds on expectation will you have to play until you pick a new color ball? ***1 point***

- 5
- 5/2
- 5/3
- 5/4
- 1

No, the answer is incorrect.
Score: 0

Accepted Answers:
5/3

12) Assume that no balls have been picked yet. On expectation, how many rounds will it take until balls of all the colors are picked? ***1 point***

- 137/60
- 5
- 137/12
- 15

No, the answer is incorrect.
Score: 0

Accepted Answers:
137/12

13) Assume that no balls have been picked yet. Your friend makes a deal with you that every time you pick a ball of a color not seen before, he'll give you 2 rupees and every time you pick a ball of a color already seen before, you need to pay him 1 rupee. Suppose you play the game until balls of all 5 colors are picked. On expectation, will you need to pay your friend money or will he need to pay you money at the end? ***1 point***

- You need to pay your friend
- Your friend needs to pay you
- The money cancels out and neither you nor your friend need to pay each other

No, the answer is incorrect.
Score: 0

Accepted Answers:
Your friend needs to pay you

For questions 14-17, consider the Quicksort algorithm to sort n distinct elements given on page 35 of the textbook “Probability and Computing: Randomized Algorithms and Probabilistic Analysis” by Michael Mitzenmacher and Eli Upfal. Also consider the discussion of the algorithm, Theorems 2.11 and 2.12, and their proofs given on pages 34-38 of the same book. We assume all elements in a given input are distinct. (Note: Please note that this material is being used for educational purposes in this assignment.)

14) Suppose the input is probabilistic in that the input list is ordered uniformly at random as discussed on page 36. Does it now help to implement Quicksort with random pivot choices at each step instead of choosing the pivot deterministically? ***1 point***

- Deterministic Quicksort takes less expected number of comparisons compared to just choosing the pivot at random.
- Deterministic Quicksort takes more expected number of comparisons compared to just choosing the pivot at random
- Both deterministic and randomized Quicksort implementations require the same number of comparisons on expectation.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Both deterministic and randomized Quicksort implementations require the same number of comparisons on expectation.

15) Suppose that for Quicksort we deterministically always chose the middle element in the input list as the pivot. Now suppose that we are told that the set of all inputs (lists) that we will ever get for the algorithm has the following property: any given input list will be divided into two roughly equal size lists (say, at most different by a small constant) at every step of the recursion using this pivot choice strategy. ***1 point***

Now, what is the worst case running time of deterministic Quicksort with the given pivot choice strategy for such a scenario?

- $O(n^2)$
- $O(nlogn)$
- $O(n)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$O(nlogn)$

16) Suppose that for Quicksort we deterministically always chose the middle element in the input list as the pivot. Now suppose that we are told that the set of all inputs (lists) that we will ever get for the algorithm has the following property: any given list will be divided into lists of size roughly 1/3rd and 2/3rd of the original list size at every step of the recursion using this pivot choice strategy. Now, what is the worst case running time of deterministic Quicksort with the given pivot choice strategy for such a scenario? ***1 point***

- $O(n^2)$
- $O(nlogn)$
- $O(n)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$O(nlogn)$

17) Suppose that for Quicksort we deterministically always chose the middle element as the pivot. Now suppose that we are told that the set of all inputs (lists) that we will ever get for the algorithm has the following properties: (i) for all but one of those inputs, the input list is divided into two roughly equal size lists at every step of the recursion using this pivot choice strategy and (ii) for the remaining input, the list is divided at every stage of the recursion such that one element goes into one list and the remaining elements (minus the pivot) go into the other list using this pivot choice strategy. Now, what is the worst case running time of deterministic Quicksort with the given pivot choice strategy for such a scenario? ***1 point***

- $O(n^2)$
- $O(nlogn)$
- $O(n)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$O(n^2)$

18) Let X be a discrete random variable. Is the following inequality correct or incorrect? ***1 point***

$E[X^2+2X-5]<(E[X])^2+2E[X]-5?$

- Correct
- Incorrect
- Not enough information to say

No, the answer is incorrect.
Score: 0

Accepted Answers:
Incorrect

19) Let X be a discrete random variable. Is the following inequality correct or incorrect? ***1 point***

$E[2X-5]<2E[X]-5$

- Correct
- Incorrect
- Not enough information to say

No, the answer is incorrect.
Score: 0

Accepted Answers:
Incorrect

20) Consider two unweighted dice with numbers from 1 to 6 on each of the faces of the two dice. ***1 point***
Consider the event of rolling both dice and taking the numbers that appear on the top face of each die. Let X be a random variable denoting the sum of those numbers and let Y be a random variable denoting the product of those numbers.
What is the value of E[E[X | Y]]?

- 11/3
- 11/2
- 6
- 7

No, the answer is incorrect.
Score: 0

Accepted Answers:
7

Previous Page End