

Week 8 Solutions

1. What is the efficiency of Dijkstra algorithm (as discussed in class) using standard binary heaps? Assume that $E \ll V$.
 - A. $O(E \log V)$
 - B. $O(|E| + |V|)$
 - C. $O(V \log E)$
 - D. $O(E + V^2)$

Answer: A

The efficiency depends on the number of DeleteMins ie. V and E updates for priority queues that were use.

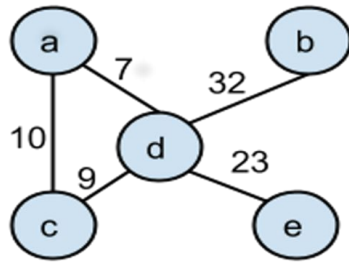
2. What is the efficiency of Dijkstra algorithm (as discussed in class) using standard array?
 - A. $O(E \log V)$
 - B. $O(|E| + |V|)$
 - C. $O(V \log E)$
 - D. $O(E + V^2)$

Answer: D

3. The minimum spanning tree will be useful in which of the following scenarios:
 - A. Telecommunications company laying cable to a new neighborhood.
 - B. By an airline laying out flight routes.
 - C. By an architect to lay out corridors between offices in a new office building
 - D. None of the Above
 - E. All of the Above

Answer: A

4. What are the total number of spanning trees possible for the given graph:



- A. 3
- B. 2
- C. 1
- D. None of above

Answer: A

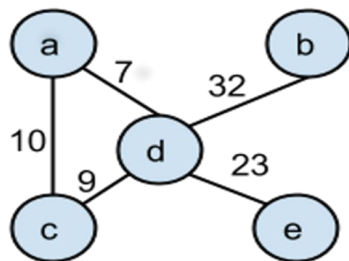
5. An undirected graph G has n nodes. Its adjacency matrix is given by an $n \times n$ square matrix whose
- (i) diagonal elements are 0's and
 - (ii) non-diagonal elements are 1's.

Which one of the following is TRUE?

- A. Graph G has no minimum spanning tree (MST)
- B. Graph G has a unique MST of cost $n-1$
- C. Graph G has multiple distinct MST of cost $n-1$
- D. None of the above

Answer: C

6. What is the weight of the minimum spanning tree of the graph given below:



- A. 71
- B. 72
- C. 74
- D. 75

Answer: A

7. Consider a graph $G(V,E)$ where V is set of vertices and E is the set of edges represented using adjacency list. What is the running time of Prim's algorithm using priority queue on the graph G ?
- A. $O(E \log V)$
 - B. $O(V \log E)$
 - C. $O(V+E)$
 - D. None of the above

Answer: A

8. Which of the following hash functions will distribute keys more uniformly over 10 buckets numbered 0 to 9 for i ranging from 0 to 1000?
- A. $i^2 \bmod 10$
 - B. $i^3 \bmod 10$
 - C. $10*i \bmod 10$

Answer: B

$10i \bmod 10$ will lead everything to same bucket

i	i^2	i^3
0	0	0
1	1	1
2	4	8
3	9	7
4	6	4
5	5	5
6	6	6
7	9	3
8	1	2

9	9	9
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Numbers distributed uniformly if we use cubic function

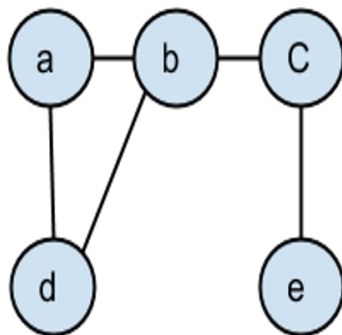
9. For an undirected graph G. If number of vertices is 100 and number of edges is 300. The minimum cost spanning tree has a weight of 500. . If the weight of each edge in graph G is increased by five, the weight of the minimum spanning tree becomes:

- A. 500
- B. 1000
- C. 1005
- D. 995

Answer: D

As there will be 99 edges in Minimum Spanning Tree (MST). When weight of every edge is increased by 5, the increment in weight of MST is = $99 * 5 = 495$ So new weight of MST is $500 + 495$ which is 995.

10. List the articulation points in the graphs given below:



- A. b
- B. b & e
- C. b & c
- D. b & d

Answer: C

11. The following input is given to a hash function

$$h(x) = x \bmod 10:$$

22, 34, 71, 79, 89, 11, 73, 99

Which of the following statements are true?

- A. 79, 89 and 99 hash to the same value

- B. 71 and 11 hash to different values
- C. All elements hash to the same value
- D. Each element hashes to a different value

Answer: A

They all hash to same value 9

12. Given a hash table with 100 slots that stores 1000 elements. What is the load factor (α) for the table
- A. .01
 - B. .1
 - C. 1
 - D. 10

Answer: D

Load factor is Load factor is (number of elements/number of slots)

13. Consider a hash table of size 7 (starting index 0) with one bucket per slot and the following hash function $(3x+4) \bmod 7$.
if elements 1,3,8,10 are inserted in the hash table sequentially using open addressing (closed hashing); What will be the index of element 10.
In open addressing, a hash collision is resolved by searching through next alternate locations until an unused slot is found.
- A. 0
 - B. 1
 - C. 2
 - D. 3

Answer: C

Hash table will look like this after inserting all the four elements.

0 - 1

1 - 8

2 - 10

3 -

4 -

5 -

14. In a weighted graph, assume that the shortest path from a source 's' to a destination 't' is correctly calculated using a shortest path algorithm. Is the following statement true?

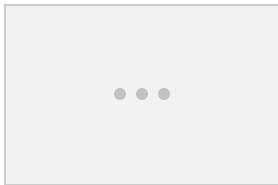
If we increase the weight of every edge by 1, the shortest path always remains same

- A. Yes
- B. No

Answer: B

Counter Example:

In the below graph the shortest distance from 's' to 'c' is 3 (through nodes a,b). Now increasing each edge weight by 1 will make shortest distance as 5 rather than 6



15. Is the following statement valid about shortest paths?

Given a graph, shortest path from a source to all other vertices is calculated. If we modify the graph such that weights of all edges is double of the original weight, then the shortest path remains same only the total weight of path changes.

Assuming that all the edge weights are positive.

- A. Yes
- B. No

Answer: A

The shortest path remains same. It is like if we change unit of distance from meter to kilometer, the shortest paths don't change.

16. Given a weighted graph where weights of all edges are unique (no two edge have same weights), there is always a unique shortest path from a source to destination in such a graph.

- A. Yes
- B. No

Answer: B

In the graph below, even with different weights shortest path is not unique between s and t. Two path are possible

$s \rightarrow a \rightarrow t$ and $s \rightarrow t$

