Week 8: Assignment 8

The due date for submitting this assignment has passed.

Due on 2021-10-20, 23:59 IST.

As per our records you have not submitted this assignment.

1) Let $b_1, \ldots, b_n$ be n numbers, each chosen independently, uniformly at random from the set {-1,+1}. Let $X = \sum_{i,j \in [n]} b_i \cdot b_j$ be a random variable, where [n] denotes the set {1,...,n}. Then, what is the expected value of X? *1 point*

- n.
- n^2.
- $\sqrt{n}$.
- 0.

No, the answer is incorrect.
Score: 0
Accepted Answers:
n.

2) Let $v_1, \ldots, v_n$ be any n unit vectors in $\mathbb{R}^n$. Then which of the following is true? *1 point*

- There exists a binary vector $b \in \{-1, 1\}^n$ such that $\|\sum_{i \in [n]} b_i \cdot v_i\| \leq \sqrt{n}$.
- There exists a binary vector $b \in \{-1, 1\}^n$ such that $\|\sum_{i \in [n]} b_i \cdot v_i\| \geq \sqrt{n}$.
- Both of the above.
- None of the above.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Both of the above.

3) Let a class of students have 30 boys and 20 girls. For the morning assembly, suppose the class teacher forms a line of all the students in a random order. What is the probability that all the girls stand before boys in the line? *1 point*

- $\frac{1}{50!}$
- $\frac{\binom{50}{20}}{50!}$
- $\frac{1}{\binom{50}{20}}$
- $\frac{20}{50}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{1}{\binom{50}{20}}$

4) Consider a stream of m distinct elements $\sigma = \langle a_1, a_2, \ldots, a_m \rangle$, where each $a_i \in [n]$. Let $h : [n] \to [n]$ be randomly chosen from a family of pairwise independent hash functions. For each $i \in [m]$ and some fixed integer $r \in [n]$, define random variable $Y_{r,i} = 1$, if $h(a_i) \geq r$ and 0 otherwise. Define random variable $X_r = \sum_{i \in [m]} Y_{r,i}$. Then, which of the following is true? *1 point*

- $\text{var}(X_r) = \sum_{i \in [m]} \text{var}(Y_{r,i})$.
- $E[Y_{r,i}^2] = E[Y_{r,i}]$ for all $i \in [m]$.
- $\text{var}(X_r) \leq E[X_r]$.
- All of the above.

No, the answer is incorrect.
Score: 0
Accepted Answers:
All of the above.

5) Consider the following stream of 50 elements $\sigma = \langle 1, 1, 2, 2, \ldots, 25, 25 \rangle$. Let $h : \{1, \ldots, 25\} \to \{-1, +1\}$ be randomly chosen from a family of pairwise independent hash functions. Let a_i denote the i-th token in stream. Let random variable $Y = Z^2$, where $Z = \sum_{i \in [50]} h(a_i)$. Then, what is the expected value of Y? *1 point*

- 0.
- 100.
- 50.
- Cannot determine as h is not from a family of 4-wise independent hash functions.

No, the answer is incorrect.
Score: 0
Accepted Answers:
100.