

## Course outline

How does an NPTEL online course work?

Week 0

Week 1

Week 2

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Week 8

● Lecture 29: Extremal Set Families

● Lecture 30: Super Concentrators

● Lecture 31: Streaming Algorithms I

● Lecture 32: Streaming Algorithms II

● Week-8 Slides: Extremal set families and super concentrators

● Week-8 Slides: Streaming algorithms

○ **Quiz: Week 8: Assignment 8**

○ Week 8: Assignment 8 Solutions

● Feedback For Week 8

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# Week 8: Assignment 8

The due date for submitting this assignment has passed.

**Due on 2021-10-20, 23:59 IST.**

As per our records you have not submitted this assignment.

1) Let  $b_1, \dots, b_n$  be  $n$  numbers, each chosen independently, uniformly at random from the set  $\{-1, +1\}$ . Let  $X = \sum_{i,j \in [n]} b_i \cdot b_j$  be a random variable, where  $[n]$  denotes the set  $\{1, \dots, n\}$ . Then, what is the expected value of  $X$ ? **1 point**

- $n$ .
- $n^2$ .
- $\sqrt{n}$ .
- $0$ .

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $n$ .

2) Let  $v_1, \dots, v_n$  be any  $n$  unit vectors in  $\mathbb{R}^n$ . Then which of the following is true? **1 point**

- There exists a binary vector  $b \in \{-1, 1\}^n$  such that  $\|\sum_{i \in [n]} b_i \cdot v_i\| \leq \sqrt{n}$ .
- There exists a binary vector  $b \in \{-1, 1\}^n$  such that  $\|\sum_{i \in [n]} b_i \cdot v_i\| \geq \sqrt{n}$ .
- Both of the above.
- None of the above.

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
Both of the above.

3) Let a class of students have 30 boys and 20 girls. For the morning assembly, suppose the class teacher forms a line of all the students in a random order. What is the probability that all the girls stand before boys in the line? **1 point**

- $\frac{1}{50!}$
- $\frac{\binom{50}{20}}{50!}$
- $\frac{1}{\binom{50}{20}}$
- $\frac{20}{50}$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $\frac{1}{\binom{50}{20}}$

4) Consider a stream of  $m$  distinct elements  $\sigma = \langle a_1, a_2, \dots, a_m \rangle$ , where each  $a_i \in [n]$ . Let  $h : [n] \rightarrow [n]$  be randomly chosen from a family of pairwise independent hash functions. For each  $i \in [m]$  and some fixed integer  $r \in [n]$ , define random variable  $Y_{r,i} = 1$ , if  $h(a_i) \geq r$  and 0 otherwise. Define random variable  $X_r = \sum_{i \in [m]} Y_{r,i}$ . Then, which of the following is true? **1 point**

- $\text{var}(X_r) = \sum_{i \in [m]} \text{var}(Y_{r,i})$ .
- $E[Y_{r,i}^2] = E[Y_{r,i}]$  for all  $i \in [m]$ .
- $\text{var}(X_r) \leq E[X_r]$ .
- All of the above.

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
All of the above.

5) Consider the following stream of 50 elements  $\sigma = \langle 1, 1, 2, 2, \dots, 25, 25 \rangle$ . Let  $h : \{1, \dots, 25\} \rightarrow \{-1, +1\}$  be randomly chosen from a family of pairwise independent hash functions. Let  $a_i$  denote the  $i$ -th token in stream. Let random variable  $Y = Z^2$ , where  $Z = \sum_{i \in [50]} h(a_i)$ . Then, what is the expected value of  $Y$ ? **1 point**

- 0.
- 100.
- 50.
- Cannot determine as  $h$  is not from a family of 4-wise independent hash functions.

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
100.