Week 7: Assignment 7

The due date for submitting this assignment has passed.

Due on 2021-10-13, 23:59 IST.

As per our records you have not submitted this assignment.

1) Let x, x' be two distinct 20-bit binary strings and y,y' be two 10-bit binary strings. Then which of the following is true for pairwise independent hashing: $\Phi_R : \{0,1\}^{20} \rightarrow \{0,1\}^{10}$? ***1 point***

- $P(\Phi_R(x) = y \wedge \Phi_R(x') = y') = \frac{1}{100}$
- $P(\Phi_R(x) = y \wedge \Phi_R(x') = y') = \frac{1}{400}$
- $P(\Phi_R(x) = y \wedge \Phi_R(x') = y') = \frac{1}{2^{20}}$
- $P(\Phi_R(x) = y \wedge \Phi_R(x') = y') = \frac{1}{2^{10}}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$P(\Phi_R(x) = y \wedge \Phi_R(x') = y') = \frac{1}{2^{20}}$

2) Let x, x' be two distinct elements of a set S. Then which of the following is true for pairwise independent hashing: $\Phi_R : S \rightarrow T$? ***1 point***

- $P(\Phi_R(x) = \Phi_R(x')) = \frac{1}{|T|^2}$
- $P(\Phi_R(x) = \Phi_R(x')) = \frac{1}{|T|}$
- $P(\Phi_R(x) = \Phi_R(x')) = \frac{1}{|S|^2}$
- $P(\Phi_R(x) = \Phi_R(x')) = \frac{1}{|S|}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$P(\Phi_R(x) = \Phi_R(x')) = \frac{1}{|T|}$

3) Let K_n denote a complete undirected graph on n vertices. Suppose you are given K_5 and K_7 where each edge is randomly colored with RED or BLUE in both K_5 and K_7. Now, suppose I pick out 3 distinct vertices each, in both of the graphs. What is the probability that triangle formed by the 3 vertices in K_5 is monochromatic? What is the probability that triangle in K_7 is monochromatic, respectively? ***1 point***

- 1/5, 1/7
- 3/5, 3/7
- 1/4, 1/4
- 1/16, 1/64

No, the answer is incorrect.
Score: 0

Accepted Answers:
1/4, 1/4

4) Consider an IPL cricket tournament consisting of 8 teams. Suppose every season each team plays every other team exactly once. For each season, we can define a directed tournament graph T of 8 vertices, where each team corresponds to a vertex. There is a directed edge from vertex x to vertex y if team y beats x. A Hamiltonian path in a graph is defined as a path which visits each vertex exactly once. A Hamiltonian path in this tournament graph looks like $(x_1, \ldots, x_8)$, where there is an edge from $x_i \rightarrow x_{i+1}$ for each $1 \leq i \leq 7$. Suppose IPL is organized fairly and all teams are balanced to ensure that in every match both teams have equal chance of winning. Given any sequence of 8 teams, what is the probability that the sequence forms a Hamiltonian path in T? ***1 point***

- 1/128
- 1/256
- 1/8
- 1/7

No, the answer is incorrect.
Score: 0

Accepted Answers:
1/128

5) Lets continue the previous question. Consider all possible distinct tournament graphs on 8 vertices. Suppose every year a new season of IPL is played till all the tournament graphs on 8 vertices are exhausted. Further suppose that every season of IPL is fair, i.e. both teams have equal chance of winning in every match for each season. Then, which of the following is true? ***1 point***
[Hint: Every tournament graph T is random. Define an indicator random variable X_π which takes value 1 if π is a Hamiltonian path in T and 0 otherwise. Also consider a random variable X which denotes the total number of Hamiltonian paths in T. Calculate expected value of X.]

- There exists an IPL season for which its tournament graph has at least $8!/2^7$ Hamiltonian paths.
- There exists an IPL season for which its tournament graph has only 1 Hamiltonian path.
- Both of the above.
- None of the above.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Both of the above.