

## Course outline

How does an NPTEL online course work?

Week 0

Week 1

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Week 4

• Lecture 11: More on Polynomial Hierarchy

• Lecture 12: Alternating Turing Machines

• Lecture 13: Equivalence of Quantifier and Oracle Based Definitions of Polynomial Hierarchy

 Quiz : Assignment 4

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 Assignment 4 Solution

week 5

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# Assignment 4

The due date for submitting this assignment has passed.

**Due on 2021-02-17, 23:59 IST.**

As per our records you have not submitted this assignment.

- 1) A language  $L$  is in the class  $DP$  iff there are two languages  $L_1 \in NP$  and  $L_2 \in coNP$  such that  $L = L_1 \cap L_2$

**4 points**

Consider the following problems:

- $SAT - UNSAT$ :  $\{(\phi, \phi') \mid \phi \text{ is satisfiable and } \phi' \text{ is not satisfiable}\}$
  - $ALMOST - SAT$ :  $\{\phi \mid \phi \text{ is unsatisfiable, but deleting any clause makes it satisfiable}\}$
- Which of the following option(s) is(are) true?

- Only  $SAT - UNSAT$  is in  $DP$
- Only  $ALMOST - SAT$  is in  $DP$
- Both  $SAT - UNSAT$  and  $ALMOST - SAT$  are in  $DP$
- Neither of  $SAT - UNSAT$  or  $ALMOST - SAT$  is in  $DP$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
Both  $SAT - UNSAT$  and  $ALMOST - SAT$  are in  $DP$

- 2) Define  $P_{||}^{NP}$  to be the set of languages decided by an oracle machine working as follows:

**6 points**

Given input  $x$ , the machine computes in polynomial time a polynomial number of non adaptive  $NP$  queries and receives all the correct answers at once. Then it decides whether  $x \in L$  in polynomial time.

Define  $P^{NP[\log n]}$  to be the class of all languages decided by a polynomial time oracle machine which on input  $x$ , asks a total of  $O(\log(|x|))$  many  $NP$  queries (possibly adaptive) and decides whether  $x \in L$ .

Mark the correct option(s)

- $P_{||}^{NP} \subseteq P^{NP[\log n]}$
- $P_{||}^{NP} \subsetneq P^{NP[\log n]}$
- $P^{NP[\log n]} \subseteq P_{||}^{NP}$
- $P^{NP[\log n]} \subsetneq P_{||}^{NP}$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $P_{||}^{NP} \subseteq P^{NP[\log n]}$   
 $P^{NP[\log n]} \subseteq P_{||}^{NP}$

- 3) Which of the following statement(s) is/are known to be true?

**2 points**

- The polynomial hierarchy does not have a complete problem
- If  $EXP$  is in  $PH$ , then  $PH$  collapses
- If  $\Sigma_i^P$  has a complete problem for some  $i$ , then  $PH$  collapses
- It is not known whether  $PSPACE$  is in  $PH$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
If  $EXP$  is in  $PH$ , then  $PH$  collapses  
It is not known whether  $PSPACE$  is in  $PH$

- 4) Which of the following statements are known to be true?

**2 points**

- $P = \bigcup_{i \in \mathbb{N}} \Sigma_i^P$  unless  $P \neq NP$
- $P = AP$  unless  $P \neq NP$
- $PH$  collapses unless  $AP \neq PH$
- $PSPACE \neq APSPACE$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $P = \bigcup_{i \in \mathbb{N}} \Sigma_i^P$  unless  $P \neq NP$   
 $PH$  collapses unless  $AP \neq PH$

- 5) Suppose a language  $L \in NP \cap coNP$ . Then which of the following is/are true?

**4 points**

- $L \notin P$
- $L \in P$
- $P^L \in NP$
- $P^L \in coNP$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $P^L \in NP$   
 $P^L \in coNP$

- 6) Consider a modified version of Traveling Salesman problem in which, given a graph  $G$  and a cost  $c$ , you have to decide whether the shortest tour in the graph has cost that is exactly equal to  $c$ . Recall that a tour is a hamiltonian cycle of the graph and its cost is the sum of costs of each edge in the cycle.

**2 points**

What is the smallest complexity class that contains this problem?

- $NP$
- $coNP$
- $\Sigma_2$
- $\Pi_2$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $\Sigma_2$

- 7) Consider the polynomial space hierarchy  $SH$ :

**4 points**

$\Sigma_0^S = \Pi_0^S = PSPACE$ . For all  $i \geq 0$ ,

$\Sigma_{i+1}^S = NPSPACE^{\Sigma_i^S}$

$\Pi_i^S = co\Sigma_i^S$

Finally,  $SH = \bigcup_i \Sigma_i^S$

Mark the correct statement(s) about  $SH$ :

- There exists a complete problem for  $SH$
- There is no known complete problem for  $SH$
- Every level of  $SH$  has a complete problem
- $SH \subseteq PSPACE$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
There exists a complete problem for  $SH$   
Every level of  $SH$  has a complete problem  
 $SH \subseteq PSPACE$

- 8) Suppose  $\overline{NP}^{NP^{NP}} = NP^{NP}$  Then what can we say about  $PH$ ?

**2 points**

- $PH$  collapses to  $P$
- All the levels of  $PH$  cannot be distinct.
- If  $PH$  collapses then necessarily  $P = NP$
- $PSPACE$  would be in  $PH$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
All the levels of  $PH$  cannot be distinct.