# CS 746: Riemann Hypothesis and Its Applications
# Practice Problem Set 1

1. Show that all the rational functions ($\frac{p(z)}{q(z)}$, for some polynomial $p(z)$ and $q(z)$) are analytic over a domain in which $q(z) \neq 0$ at every point.

2. Show that $f(z) = e^{iz^2}$ is an entire function.

3. Study the analyticity of the following functions: $e^z$ and $sin(z) = \frac{e^{iz} - e^{-iz}}{2i}$.

4. Work out the relationship between absolute convergence and uniform convergence.

5. Let $f$ be a power series with radius of convergence $R$, then show that for any $z$ such that $|z| > R$, $f$ is absolutely divergent.

6. Show that given the absolutely convergent series

$$A = \sum_{n=0}^{\infty} \alpha_n, \; B = \sum_{n=0}^{\infty} \beta_n$$

we have the absolutely convergent series

$$AB = \sum_{n=0}^{\infty} \gamma_n, \; \gamma_n = \sum_{j=0}^{n} \alpha_j \beta_{n-j}.$$

7. If $D$ be a domain bounded by a contour $C$ for which Cauchy's theorem is valid and $f$ is continuous on $C$ and regular (analytic and single-valued) in $D$, then show that $|f| \leq M$ on $C$ implies $|f| \leq M$ in $D$ and if $|f| = M$ in $D$, then $f$ is a constant.
(Hints: Apply Cauchy's Integral Formula to $f(z)^n$ for the first part and to $\frac{d}{dz}[f(z)^n]$ for the second part)