Unit 10 - Week 8: Graph Theoretic Algorithms

Assignment 8

The due date for submitting this assignment has passed. **Due on 2020-03-25, 23:59 IST.**
As per our records you have not submitted this assignment.

1) Let G be the graph shown below. Questions 1-4 are based on G. Let A be the adjacency matrix of G. i.e, $A_{ij} = 1$ iff (i, j) is an edge of G. Let D be the distance matrix of G; i.e, $D_{ij} = k$ iff the shortest path between i and j is of length k. Let G' be the graph obtained by placing an edge between every pair of vertices $i \neq j$ that are at distance 1 or 2 in G.Let A' be the adjacency matrix of G' and let D' be the distance matrix of G'. Let $Z = A^2$(The matrix multiplication considered is integer matrix multiplication). **1 point**

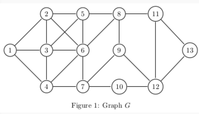

Figure 1: Graph G

Which of the following statements are true?

- G' is a bipartite graph.
- The vertex set $\{1, 2, 3, 4, 5, 6, 7\}$ form a clique in G'.
- The vertex set $\{2, 3, 4, 5, 6, 7, 8, 9, 10\}$ form a clique in G'
- The vertex set $\{8, 9, 10, 11, 12, 13\}$ form a clique in G'

No, the answer is incorrect.
Score: 0
Accepted Answers:
The vertex set $\{1, 2, 3, 4, 5, 6, 7\}$ form a clique in G'.

2) Which of the following statements are true? **1 point**

- $Z = A'$
- The diagonal entries of Z are all 1.
- If (i, j) is an edge of G, then $Z_{ij} = 1$.
- If (i, j) and (j, k) are edges of G, then $Z_{ik} \geq 1$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If (i, j) and (j, k) are edges of G, then $Z_{ik} \geq 1$.

3) Which of the following statements are true? **1 point**

- $D'_{113} = 3$
- $D'_{612} = 1$
- $D_{113} = 5$
- For every pair of vertices (i, j) other than $(1, 13)$, D'_{ij} is less than or equal to 2.
- D_{ij} is less than or equal to 5 for every pair of vertices i and j.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$D_{113} = 5$
For every pair of vertices (i, j) other than $(1, 13)$, D'_{ij} is less than or equal to 2.
D_{ij} is less than or equal to 5 for every pair of vertices i and j.

4) Which pairs of vertices given below satisfy the relation $D_{ij} = 2D'_{ij}$? **1 point**

- $i = 1, j = 3$
- $i = 6, j = 9$
- $i = 1, j = 9$
- $i = 2, j = 13$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$i = 6, j = 9$
$i = 2, j = 13$

5) Let G_1 be a graph and i and j be vertices of G. Let D be the distance matrix of G and let $D_{ij} = 1000$. Let k be any neighbor of i. Which of the following statements are true? **1 point**

- If $D_{kj} \equiv 2 \mod 3$, then there is a shortest path from i to $j$passing k.
- $D_{kj} = 999$ or $D_{kj} = 1001$
- If $D_{kj} \equiv 1 \mod 3$, then there is a shortest path from i to j passing k.
- If $D_{kj} \equiv 0 \mod 3$, then there is a shortest path from i to j passing k.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $D_{kj} \equiv 0 \mod 3$, then there is a shortest path from i to j passing k.

6) Consider the Boolean matrices A and B given below; **1 point**

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let P be Boolean product of matrix A and B and let W be any witness matrix corresponding to P. Which of the following statements are true?

- $W_{11} = 2$
- $W_{12} = W_{22} = 0$
- $W_{34} = 2$
- $W_{44} = 1$ or $W_{44} = 4$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$W_{11} = 2$
$W_{12} = W_{22} = 0$
$W_{44} = 1$ or $W_{44} = 4$

7) Let A and B be 100×100 matrices. Let P denote the Boolean product of A and B. Let R_1 be a set random subset (chosen uniformly at random) of size 8. Let R_2 be a set random subset (chosen uniformly at random) of size 4. Let R_3 be a set random subset (chosen uniformly at random) of size 2. Which of the following statements are true? **1 point**

- For any entry x of P having at least 7 witnesses and at most 12 witnesses, there is at least a $\frac{1}{5^2}$ probability that the set R_1 will contain exactly one witness for x
- For any entry $x$of P having at least 13 witnesses and at most 25 witnesses, there is at least a $\frac{1}{5^2}$ probability that the set R_2 will contain exactly one witness for x
- For any entry $x$of P having at least 26 witnesses and at most 50 witnesses, there is at least a $\frac{1}{5^2}$ probability that the set R_3 will contain exactly one witness for x
- None of the other choices are correct.

No, the answer is incorrect.
Score: 0
Accepted Answers:
For any entry x of P having at least 7 witnesses and at most 12 witnesses, there is at least a $\frac{1}{5^2}$ probability that the set R_1 will contain exactly one witness for x
For any entry $x$of P having at least 13 witnesses and at most 25 witnesses, there is at least a $\frac{1}{5^2}$ probability that the set R_2 will contain exactly one witness for x
For any entry $x$of P having at least 26 witnesses and at most 50 witnesses, there is at least a $\frac{1}{5^2}$ probability that the set R_3 will contain exactly one witness for x

8) Let H be the weighted graph shown below. Questions 8-11 are based on H. **1 point**

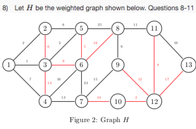

Figure 2: Graph H

Which of the following edges will be chosen as the edges of the MST during a single Boruvka phase run on the graph H?

- $(11, 13)$
- $(5, 6)$
- $(7, 9)$
- $(3, 5)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(5, 6)
(3, 5)

9) How many edges will be chosen as the edges of the MST during a single Boruvka phase run on the graph H? **1 point**

- 9
- 8
- 11
- 10

No, the answer is incorrect.
Score: 0
Accepted Answers:
10

10) In the graph Hgiven in Figure 2, consider the forest F given by the red edges. Which of the edges given below are F-heavy? **1 point**

- $(2, 5)$
- $(5, 8)$
- $(6, 7)$
- $(7, 9)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(5, 8)
(7, 9)

11) In the graph H given in Figure 2, consider the forest F given by the red edges. Which of the edges given below are F-light? **1 point**

- $(2, 5)$
- $(5, 8)$
- $(6, 7)$
- $(7, 9)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(2, 5)
(6, 7)