

NPTEL » Randomized Algorithms

AnnouncementsAbout the CourseAsk a QuestionProgressMentor

## Unit 4 - Week 2: Probability Review

Course outline
How does an NPTEL online course work?
Week 0:Prerequisite
Week 1: Introduction to Randomized Algorithms
Week 2: Probability Review <ul style="list-style-type: none"><li>Probability Review</li><li>Expectation of Random Variables</li><li>Conditional Probability and Conditional Expectation2</li><li>Birthday Paradox</li></ul>
Quiz : Assignment 2
Weekly feedback form for week 2
Week3: Moments and Deviations
Week4: Probabilistic Method
Week 5: Markov Chains
Week 6 : Markov Chains-II
Week 7: Number Theoretic Algorithms
Week 8: Graph Theoretic Algorithms
Week 9 : Approximate Counting
Week 10 : Randomization and Data Structures
Week 11 : Computational Complexity
Week 12 : Summary
Download Videos

## Assignment 2

The due date for submitting this assignment has passed.  
As per our records you have not submitted this assignment.

Due on 2020-02-12, 23:59 IST.

- 1) A coin is tossed until for the first time the same result appears twice in succession. Which of the following statements are true regarding the underlying sample space?

1 point

☐ The underlying sample space is finite.

☐ The underlying sample space consists of sequences formed by tossing a coin infinitely often.

☐ The underlying sample space is countable.

☐ The underlying sample space is uncountable.

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
The underlying sample space is countable.
- 2) Consider the sample space  $\Omega = \{HH, HT, TH, TT\}$  and the event space  $\mathcal{F} = 2^\Omega$ . Which of the following partial functions  $\mathbb{P}$  can be extended to a probability function on  $\mathcal{F}$ ?

1 point

☐  
 $\mathbb{P}(\{HH\}) = 0.01, \mathbb{P}(\{HT\}) = 0.09, \mathbb{P}(\{TH\}) = 0.09, \mathbb{P}(\{TT\}) = 0.81,$

☐  
 $\mathbb{P}(\{HH\}) = 0.01, \mathbb{P}(\{HT\}) = 0.15, \mathbb{P}(\{TH\}) = 0.03, \mathbb{P}(\{TT\}) = 0.81,$

☐  
 $\mathbb{P}(\{HH, TT\}) = 0.5, \mathbb{P}(\{HT, TH\}) = 0.5, \mathbb{P}(\{HH\}) = 0.4$

☐  
 $\mathbb{P}(\{HH, TT, TH\}) = 0.5, \mathbb{P}(\{HT, TH\}) = 0.4, \mathbb{P}(\{HH\}) = 0.4$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $\mathbb{P}(\{HH\}) = 0.01, \mathbb{P}(\{HT\}) = 0.09, \mathbb{P}(\{TH\}) = 0.09, \mathbb{P}(\{TT\}) = 0.81,$   
 $\mathbb{P}(\{HH\}) = 0.01, \mathbb{P}(\{HT\}) = 0.15, \mathbb{P}(\{TH\}) = 0.03, \mathbb{P}(\{TT\}) = 0.81,$   
 $\mathbb{P}(\{HH, TT\}) = 0.5, \mathbb{P}(\{HT, TH\}) = 0.5, \mathbb{P}(\{HH\}) = 0.4$
- 3) Consider the events  $E_1$  and  $E_2$ . Assume that the events have non zero probability. Which of the following statements are true?

1 point

☐  
If  $E_1$  and  $E_2$  are independent, then so are  $E_1^c$  and  $E_2^c$ .

☐  
 $E_1$  and  $E_1^c$  are independent

☐  
If  $E_1$  and  $E_2$  are independent, then so are  $E_1$  and  $E_2^c$ .

☐  
If  $E_1$  and  $E_2$  have a non trivial intersection, they cannot be independent.

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
If  $E_1$  and  $E_2$  are independent, then so are  $E_1^c$  and  $E_2^c$ .  
If  $E_1$  and  $E_2$  are independent, then so are  $E_1$  and  $E_2^c$ .
- 4) Let  $A, B$  and  $C$  be arbitrary events in a probability space. Let  $E$  be the event that at least two of the three events among  $A, B$ , and  $C$  occur. Which of the following expressions correspond to  $E$ ?

1 point

☐  
 $(A \cup B) \cap (B \cup C) \cap (C \cup A)$

☐  
 $(A \cap B) \cap (B \cap C) \cap (C \cap A)$

☐  
 $(A \cap B) \cup (B \cap C) \cup (C \cap A)$

☐  
 $(A \cup B) \cup (B \cup C) \cup (C \cup A)$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $(A \cup B) \cap (B \cup C) \cap (C \cup A)$   
 $(A \cap B) \cup (B \cap C) \cup (C \cap A)$
- 5) Consider throwing an unbiased dice 10 times. If it is known that the throws have produced at least one six, what is the probability of two or more sixes?

1 point

☐  
 $\frac{5^9}{6^{10}-5^{10}}$

☐  
 $1 - \frac{5^{10}}{6^{10}-5^{10}}$

☐  
 $1 - \frac{10 \cdot 5^9}{6^{10}-5^{10}}$

☐  
 $\frac{10 \cdot 5^9}{6^{10}-5^{10}}$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $1 - \frac{10 \cdot 5^9}{6^{10}-5^{10}}$
- 6) Consider two independent rolls of a standard dice. Let  $X_1$  and  $X_2$  be the numbers that appear of the first and second roll respectively. What is  $\mathbb{E}[X_1 + X_2 | X_1 = X_2]$ ?

1 point

☐ 3.5

☐ 6.5

☐ 7

☐ 7.5

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
7
- 7) Consider two independent rolls of a standard dice. Let  $X_1$  be the numbers that appear of the first and second roll respectively. Let  $Z = \max(X_1, X_2)$ . What is  $\mathbb{E}[Z]$ ?

1 point

☐ 9/2

☐ 100/36

☐ 161/36

☐ 5

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
161/36
- 8) Consider rolling an unbiased  $n$ -side dice till all the  $n$  numbers appear. Let  $X$  be the random variable denoting the number of rolls required. What is  $\mathbb{E}[X]$ ? ( $H_n$  stands for the  $n^{\text{th}}$  harmonic number.)

1 point

☐  
 $H_n$

☐  
 $\frac{H_n}{n}$

☐  
 $nH_n$

☐  
 $n \log n$

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $nH_n$
- 9) Assume that the birthday of each person is a random day chosen independently and uniformly from a 365 day year. Which of the following events is most likely?

1 point

☐ Finding two people with same birthday in a group of 23 people

☐ Finding that no two people with the same birthday in a group of 24 people

☐ Finding two people with same birthday in a group of 24 people

☐ Finding that no two people with the same birthday in a group of 23 people

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
Finding two people with same birthday in a group of 24 people
- 10) Consider a tossing a biased coin twice independently. Let  $Z$  be the random variable defined as follows; If the outcome is  $HT$  then  $Z = 1$ . If the outcome is  $TH$  then  $Z = 0$ . If the outcome is either  $HH$  or  $TT$ , repeat the tossing till an  $HT$  or  $TH$  is reached. Let  $X$  be the number of tosses required. Which of the following statements are true?

1 point

☐  
 $X$  is a geometric random variable.

☐  
 $Z$  takes values 0 and 1 with equal probability.

☐  
 $Z$  takes values 0 and 1 with a probability that depends on the bias of the coin.

☐  
 $X$  is a uniform random variable.

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $X$  is a geometric random variable.  
 $Z$  takes values 0 and 1 with equal probability.