NPTEL Online Certification

COMPUTATIONAL HYDRAULICS
Week 2 : Assignment_Solution
July 24-October 13, 2017

NOTE: Attempt **ALL** questions. Make suitable assumptions, wherever necessary.

1. Order of truncation error for Crank-Nicolson method in case of IBVP problem

- second order in space and second order in time

2. Arrange the following methods according to order of truncation error (higher to lower) for IVP
(a) Euler method (b) Fourth order Runge-Kutta (c) Modified Euler method

- b-c-a [Fourth order Runge-Kutta (4th order) Modified Euler method (3rd order) Euler method (1st order)]

3. identify the two-step IVP method

- Euler Cauchy method
- second order Runge-Kutta method

4. Identify the correct discretization for $\phi^{"}$

- $$\phi_i^{"} = \frac{\phi_i - 2\phi_{i+1} + \phi_{i+2}}{\Delta x^2} + \mathcal{O}(\Delta x)$$

  (use Taylor Series expansion)

5. Consistency is the property of

- Discretization (Mesh/grid size does not affect consistency. It is purely dependent on discretization.)

6. Accuracy of any problem depends on

- accuracy of the discretization of differential equation and boundary conditions (Only differential equation or only boundary condition does not guarantee accuracy. It depends on maximum truncation error among these two conditions.)

7. Differential equation with only spatial derivatives is called as

- boundary value problem (does not contain time derivative term. Initial condition always require time specification or time derivative.)

8. The following differential equation with a general variable ϕ

$$\frac{\partial^2\phi(x)}{\partial x^2} + \frac{\phi_0 - \phi(x)}{\lambda} = 0$$

(ϕ_0 and λ are constants) can be discretized as,

$$\frac{\phi_{i-1} - 2\phi_i + \phi_{i+1}}{\Delta x^2} + \frac{\phi_0 - \phi_i}{\lambda} = 0$$

- $$\frac{\lambda}{\Delta x^2}\phi_{i-1} - (1 + \frac{2\lambda}{\Delta x^2})\phi_i + \frac{\lambda}{\Delta x^2}\phi_{i+1} = -\phi_0$$

CONTINUED

9. Explicit or Implicit scheme depends on

- time level of space derivatives (Umbrella or inverted umbrella like structure)

10. Boundary Value Problems (BVP) can be solved as

- initial boundary value problem with arbitrary initial condition

Initial boundary value problem can be started with proper boundary condition and arbitrary initial condition. For a longer time period, time derivative will gradually vanish leading to steady state. A steady state problem and BVP will produce identical result.
Let us consider a IBVP for groundwater

$$\frac{S}{T}\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}$$

.
After consideration of a longer time period and constant boundary conditions, $\frac{S}{T}\frac{\partial h}{\partial t}$ will become zero. Thus the problem becomes essentially BVP.