

NPTEL Online Certification

COMPUTATIONAL HYDRAULICS

Week 1 : Assignment-Solution

July 24-October 13, 2017

NOTE: Attempt **ALL** questions. Make suitable assumptions, wherever necessary.

1. Motion of saltating particle is governed by force due to its submerged weight (F_G) and hydrodynamic forces, which can be solved into a lift force (F_L), a drag force (F_D) as shown in the following Figure 1.

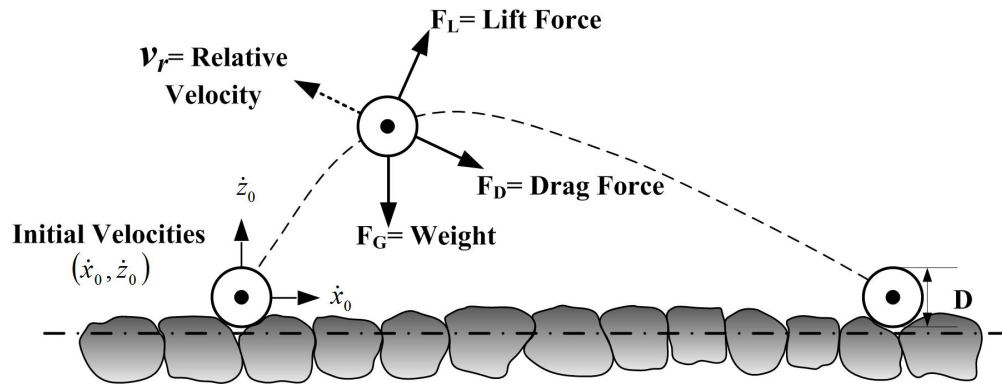


Figure 1: Definition Sketch of Particle Saltation (van Rijn, 1984)

Equations of motion for saltating particles can be expressed as,

$$m\ddot{x} - \frac{F_L}{v_r}\dot{x} - \frac{F_D}{v_r}(u - \dot{z}) = 0$$

$$m\ddot{z} - \frac{F_L}{v_r}(u - \dot{x}) + \frac{F_D}{v_r}\dot{z} + F_G = 0$$

subject to conditions,

$$x(t = 0) = x_0$$

$$z(t = 0) = z_0$$

$$\dot{x}(t = 0) = \dot{x}_0$$

$$\dot{z}(t = 0) = \dot{z}_0$$

where, m = particle mass and added fluid mass, A = particle area, u = local flow velocity, F_D = drag force, F_L = lift force, F_G = gravity force, relative particle velocity $v_r = [(u - \dot{x})^2 + \dot{z}^2]^{0.5}$. The problem can be simplified by considering, $y_1 = x$, $y_2 = z$, $y_3 = \dot{x}$, $y_4 = \dot{z}$

$$\dot{y}_1 = y_3$$

$$\dot{y}_2 = y_4$$

$$\dot{y}_3 = \frac{1}{m} \left[\frac{F_L}{v_r}y_3 + \frac{F_D}{v_r}(u - y_4) \right]$$

$$\dot{y}_4 = \frac{1}{m} \left[\frac{F_L}{v_r}(u - y_3) - \frac{F_D}{v_r}y_4 - F_G \right]$$

subject to conditions,

$$y_1(t = 0) = x_0$$

$$y_2(t = 0) = z_0$$

$$y_3(t = 0) = \dot{x}_0$$

$$y_4(t = 0) = \dot{z}_0$$

Tick the correct answer:

- (a) The equations are
 - (i)
 - (ii) Ordinary Differential Equations (**one independent variable**)
- (b) Supporting conditions for the governing equations are
 - (i) Initial Conditions (**in terms of time**)
 - (ii)
 - (iii)
- (c) The problem can be classified as
 - (i) Initial Value Problem
 - (ii)
 - (iii)

2. Let us consider a large diameter well with casing is tapping water from homogeneous isotropic artesian aquifer of uniform thickness as shown in following Figure 2.

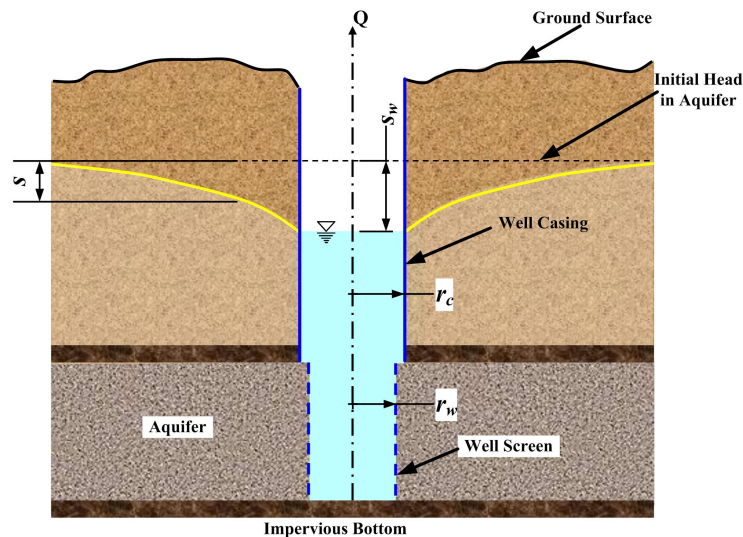


Figure 2: Idealized representation of a well open to an artesian aquifer (Papadopoulos and Cooper, 1967)

The governing equation for radial groundwater flow can be written as,

$$\frac{S}{T} \frac{\partial s}{\partial t} = \frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r}, r \geq r_w$$

subject to the conditions,

Drawdown in the aquifer at the face of the well is equal to that of the well.

$$s(r_w, t) = s_w(t)$$

Drawdown is zero at an infinite distance from the well.

$$s(\infty, t) = 0$$

Drawdown everywhere in the aquifer and in the well are initially zero.

$$s(r, 0) = 0, r \geq r_w$$

$$s_w(0) = 0$$

The rate of the discharge of the well is equal to the sum of the rate of flow of water into the well and the rate of decrease in volume of water within the well.

$$2\pi r_w T \frac{\partial s(r_w, t)}{\partial r} - \pi r_c^2 \frac{\partial s_w}{\partial t} = -Q, t > 0]$$

where

s = drawdown in the aquifer at distance r and time t , s_w = drawdown in the well at time t , r = distance from the centre of well, r_w = effective radius of well screen or open hole, r_c = radius of well casing in the interval over which the water level declines, t = time since well begins to discharge, S = coefficient of storage of aquifer, T = transmissivity of aquifer, Q = constant discharge of well.

Tick the correct answer:

- (a) The equation is
 - (i) Partial Differential Equation
 - (ii)
- (b) Supporting conditions for the governing equation are
 - (i)
 - (ii)
 - (iii) Initial and Boundary Conditions (**space and time**)
- (c) The problem can be classified as
 - (i)
 - (ii)
 - (iii) Initial Boundary Value Problem
- (d) The problem can be classified as
 - (i)
 - (ii) Time-marching Problem

3. Numerical Discretization with finite difference method and finite volume method is based on

- (a) Eulerian Approach (**all are grid based**)
- (b)
- (c)

4. In mesh-free methods, domain is discretized using

- (a)
- (b)
- (c) points (**no grids required**)

5. In well-posed problems,

- (a) solution of the problem exists
- (b) solution is unique
- (c)

References

- Papadopoulos, I. S. and Cooper, H. H. (1967). Drawdown in a well of large diameter. *Water Resources Research*, 3(1):241-244.
- van Rijn, L. C. (1984). Sediment transport, part i: Bed load transport. *Journal of Hydraulic Engineering*, 110(10):1431-1456.
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