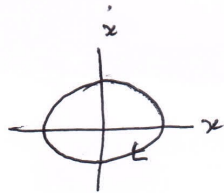


ADVANCED MATHEMATICAL METHODS FOR
CHEMISTRY - QUIZ 7 - SOLUTIONS

1



Ellipse (b)

2

$$\ddot{x} + 3\dot{x} + x = 0$$

$$x = e^{\lambda t} \Rightarrow \lambda^2 + 3\lambda + 1 = 0$$

$$\lambda = \frac{-3 \pm \sqrt{5}}{2}$$

As $t \rightarrow \infty$, larger eigenvalue dominates

$$\lambda = \frac{-3 + \sqrt{5}}{2} \approx -0.4$$

Solution $\propto e^{-0.4t}$ (b)

3

$$\ddot{x} + \gamma \dot{x} + \omega^2 x = 0$$

$$x = 0 \quad \dot{x} = 0$$

$$\frac{d}{dt} \dot{x} = \ddot{x}$$

$$\frac{d}{dt} \dot{x} = -\gamma \dot{x} - \omega^2 x$$

$$M = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -\gamma \end{bmatrix}$$

Eigenvalues

$$\lambda(\lambda + \gamma) + \omega^2 = 0$$

$$\lambda^2 + \gamma\lambda + \omega^2 = 0$$

$$\lambda = \frac{-\gamma \pm \sqrt{\gamma^2 - 4\omega^2}}{2}$$

$$\gamma = \omega \Rightarrow \lambda = -1$$

Stable. Spiral point (d)

4

$\theta = \text{odd multiple of } \pi \rightarrow \text{unstable F.P. (c)}$

5

$\theta = \text{even multiple of } \pi \rightarrow \text{stable F.P.} \rightarrow \text{steady solutions}$
Ellipses (c)

6

Equation of separatrix: $\dot{\theta} = 2\omega \cos(\theta/2) = 4\omega \cos(\theta/2)$ (c)

7

Asymptotically stable point (b)

8

$$2x - xy = 0$$

$$-y + xy = 0$$

C.P.

$$x = y = 0 \quad \text{Not interesting}$$

Say $x \neq 0$

$$y = 2 \quad x = 1$$

Say $y \neq 0$

$$x = 1 \quad y = 2$$

C.P.: (1, 2)

None of above (d)

9

$$\frac{d}{dt} x = \frac{1}{2} 0x + \dot{x}$$

$$\frac{d}{dt} \dot{x} = -3x - 4\dot{x}$$

$$\text{C.P. } \dot{x} = 0 ; x = 0$$

Both real and negative \Rightarrow

$$M = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$$

$$\lambda = -3 \text{ or } \lambda = -1$$

Stable spiral point. (b)

10

Both eigenvalues -ve \Rightarrow center (b)