ADVANCED MATHEMATICAL METHODS FOR CHEMISTRY

QUIZ 5 : SOLUTIONS

1 (a) $y'' \sin x + y' \cos x + y = 0$ is a homogeneous 2nd order linear equation

2 $y = e^{3x}\cos x \quad y' = 3y + e^{3x}\sin x \quad y'' = 9y + 6e^{3x}\sin x - y$

$$y'' - 6y' + 10y = 8y + 6\sin x\, e^{3x} - 18y - 6\sin x\, e^{3x} + 10y = 0$$

Answer (c)

3 $y' = 2\cos 2x + 1 \qquad y'' = -4\sin 2x = -4(y - x)$

$y'' + 4y - 4x = 0$ (Non homogeneous ODE).

Answer (d) - None of above

4 $y'' + \frac{y'}{\sin x} + y = 0 \qquad y_2 = u y_1 \Rightarrow \ln u = \int 2\ln y_1 + \frac{y_1'}{y_1}\, dx$

$$\ln u = \int 2\ln(1+\cos x) - \int \frac{\pm 1}{1+\cos x}\, dx = \int 2\ln(1+\cos x) - \tan(x/2)$$

Answer (b)

5 $y_h = ae^{-2x} + bxe^{-2x} \qquad W = \begin{vmatrix} e^{-2x} & xe^{-2x} \\ -2e^{-2x} & -2xe^{-2x} + e^{-2x} \end{vmatrix} = e^{-4x}$

$$u_1 = -\int \frac{xe^{-2x}\cdot 2e^{-2x}}{e^{-4x}}\, dx = -x^2 \qquad u_2 = \int \frac{e^{-2x}}{e^{-4x}} 2e^{-2x}\, dx = 2x$$

Answer (a)

6 $5^2 - 4\times 6\times 1 = 1 > 0 \Rightarrow$ overdamped H.O.

7 $a(x) = 1 - x^2 = 0$ if $x = 1$

$x = 1$ is a singular point, $x = 0$ is an ordinary point

By looking at ratios

8 $a(x) = x^2 - x \Rightarrow x = 0,\ x = 1$ are critical points

$y'' + \frac{y'}{(1-x^2)^2} + \frac{y}{x^3} = 0 \Rightarrow$ Both $x = 1$ and $x = 0$ are NOT regular c.Ps.

Answer (d)

9. $y'' - 2\alpha x y' + (2E - \alpha) y = 0$

$y = \sum_{n=0}^{\infty} a_n x^n \Rightarrow \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - 2\alpha \sum_{n=0}^{\infty} n a_n x^n + (2E-\alpha) \sum_{n=0}^{\infty} a_n x^n = 0$

Comparing x^n coeffficients

$$(n+2)(n+1) a_{n+2} - 2\alpha n a_n + (2E - \alpha) a_n = 0$$

$$\Rightarrow \frac{a_{n+2}}{a_{n+1}} = \frac{\alpha(2n+1) - 2E}{(n+1)(n+2)} \qquad (c)$$

10. $T(\phi) = \frac{1}{\sqrt{2\pi}} e^{im\phi}$ Answer (d)