Advanced Mathematical Methods for Chemistry

Quiz II Solutions

1. $\dfrac{e^{4.8/5} + e^{4.9/5} + e^{5.1/5} + e^{4.7/5} + e^{5.3/5}}{5} = 2.7$ (c)

2. $\langle x^2 \rangle = \dfrac{(4.8)^2 + (4.9)^2 + (5.1)^2 + (4.7)^2 + (5.3)^2}{5} = 24.6$ (b)

3. $\langle x^5 \rangle = \displaystyle\int_{-\infty}^{+\infty} x^5 \frac{e^{-x^2}}{\sqrt{\pi}}\, dx = 0$ (a)
(as integrand is an odd function)

4. Position $= (0.6 - 0.4) \times 50 = 10$ (None of a, b, c) (d)

5. $P(7) = \dfrac{10!}{7!\,3!} (0.7)^7 (0.3)^3 \simeq 0.25$ (c)

6. 10^{15} is a large no. Assume $n \ll 10^{15}$ $p(n) = \dfrac{(0.03t)^n}{n!} e^{-0.03t}$ (c)

Poissonian distribution

7. $e^{-x^2} \Rightarrow 2\sigma^2 = 1$ $\sigma = 1/\sqrt{2}$ (d)

8. $\langle x^4 \rangle = \dfrac{1}{\sqrt{\pi}} \displaystyle\int_{-\infty}^{+\infty} x^4 e^{-x^2} dx = \frac{2}{\sqrt{\pi}} \int_0^{\infty} x^4 e^{-x^2} dx = \frac{1}{\sqrt{\pi}} \int_0^{\infty} t^{3/2} e^{-t}\, dt$

$= \dfrac{\Gamma(5/2)}{\sqrt{\pi}} = \dfrac{3}{2} \times \dfrac{1}{2} \times \dfrac{\sqrt{\pi}}{\sqrt{\pi}} = \dfrac{3}{4}$ (c)

9. $\langle x p_x \rangle = 2 \displaystyle\int_0^2 \sin(2\pi x)\; x \cdot -i\hbar \frac{\partial}{\partial x} (\sin 2\pi x)\, dx$

$= \pi \displaystyle\int_0^2 \sin(4\pi x)\; -i\hbar x\, dx = \frac{i\hbar}{2}$ (d)

10. $\langle V_x^2 V_y^2 \rangle = \langle V_x^2 \rangle \langle V_y^2 \rangle = 10R \times 10R = 100R^2$ (c)