Unit 13 - Week 11 - Importance of Density Matrix in Quantum Computing Implementation

Assignment-11

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-10-16, 23:59 IST.

1) The density matrix for a qubit state $|\psi\rangle = c|0\rangle + d|1\rangle$ is defined as: $\rho = |\psi\rangle\langle\psi|$. What are its the eigenvectors and eigenvalues? *1 point*

- The eigenvalues of $\rho = \begin{pmatrix} |c|^2 & cd^* \\ dc^* & |d|^2 \end{pmatrix}$ are 1 and 0 and the corresponding eigenvectors are: $\begin{pmatrix} c \\ d \end{pmatrix}, \begin{pmatrix} d \\ -c^* \end{pmatrix}$.
- The eigenvalues of ρ are c and d and the corresponding eigenvectors are: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}$.
- The eigenvalues of $\rho = \begin{pmatrix} c^2 & cd^* \\ dc^* & d^2 \end{pmatrix}$ are 0 and 1 and the corresponding eigenvectors are: $\begin{pmatrix} c \\ d \end{pmatrix}, \begin{pmatrix} d^* \\ c^* \end{pmatrix}$.
- none of the above

No, the answer is incorrect.
Score: 0
Accepted Answers:
The eigenvalues of $\rho = \begin{pmatrix} |c|^2 & cd^* \\ dc^* & |d|^2 \end{pmatrix}$ are 1 and 0 and the corresponding eigenvectors are: $\begin{pmatrix} c \\ d \end{pmatrix}, \begin{pmatrix} d \\ -c^* \end{pmatrix}$.

2) Consider the density matrix $\sigma = \frac{1}{2x}(|0\rangle\langle 0| + |1\rangle\langle 1|)$, where '$x$' can be any non-zero complex number. Which is the correct statement for eigenvectors and eigenvalues of σ? *1 point*

- The eigenvalues of σ are both ½ and the eigenvectors are $|0\rangle, |1\rangle$ only for all 'x'.
- The eigenvalues of σ are 1 and 0 and the eigenvectors are $|0\rangle, |1\rangle$ or any two orthogonal vectors in this Hilbert space that contains 'x'.
- The eigenvalues of σ are -1 and 0 for $x = 1$ and the corresponding eigenvectors are $|0\rangle, |1\rangle$.
- If $x = 1$, both the eigenvalues of σ are ½ and the eigenvectors are $|0\rangle, |1\rangle$ or any two orthogonal vectors in this Hilbert space.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $x = 1$, both the eigenvalues of σ are ½ and the eigenvectors are $|0\rangle, |1\rangle$ or any two orthogonal vectors in this Hilbert space.

3) If $\rho = ((\langle 0|a^* + \langle 1|b^*)(a|0\rangle + b|1\rangle))$ and $\sigma = \frac{1}{2x}(|0\rangle\langle 0| + |1\rangle\langle 1|)$, compare the Traces of σ^2 and ρ^2. *1 point*

- Trace (σ^2) > Trace (ρ^2) for all possible values of a, b, and x
- Trace (σ^2) = ½ Trace (ρ^2), only when a=b=x=1
- Trace (σ^2) = ½Trace (ρ^2) for all possible values of a, b, and as long as $x \neq 0$
- no relation exist between two traces

No, the answer is incorrect.
Score: 0
Accepted Answers:
Trace (σ^2) = ½ Trace (ρ^2), only when a=b=x=1

4) For a system with total angular momentum 1, if its system ensemble is described with the density matrix, $\rho = \frac{1}{4}\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, which of the following is false? *1 point*

- ρ is a Hermitian matrix that can undergo a trace preserving transformation of basis state to one in which ρ is diagonal
- ρ represents a mixed state and is not a pure state.
- Expectation value of ρ with respect to any state is negative
- none of the above

No, the answer is incorrect.
Score: 0
Accepted Answers:
Expectation value of ρ with respect to any state is negative

5) The entropy, S, corresponding to density matrix $\rho = \begin{pmatrix} \theta & 0 \\ 0 & 1-\theta \end{pmatrix}$ reaches maximum when the probability of being in either state is ½. This means *1 point*

- the entropy for a pure state, with θ = 1 or θ = 0, is zero.
- as the state becomes "less pure", there is minimal "knowledge" about the state.
- the choice basis set for the density matrix is such that $0 \le \theta \le 1$ is the probability that the system is in state 1.
- the entropy calculation: $S = -Trace(\rho ln\rho)$ is incorrect.

No, the answer is incorrect.
Score: 0
Accepted Answers:
as the state becomes "less pure", there is minimal "knowledge" about the state.

6) For a density matrix ρ, Trace $(\rho^2) = 1$ and entropy, $S(\rho) = 1$ are equivalent statements, if and only if *1 point*

- information content of mixed state is measured in terms of the entropy of the system rather than the density matrix of the system
- entropy of the system increases
- the density matrix ρ is equal to its negative square: $-\rho^2$.
- ρ has a single eigenvalue of 1 while all its other eigenvalues are 0.

No, the answer is incorrect.
Score: 0
Accepted Answers:
ρ has a single eigenvalue of 1 while all its other eigenvalues are 0.

7) In terms of density matrix, ρ, which of the following is the correct representation of the expectation value for an observable operator, $\hat{A}$? *1 point*

- $\langle A\rangle = Tr(\rho A)$ if and only if ρ is mixed state
- $\langle A\rangle = Tr(\rho^2 A)$ irrespective of whether ρ is pure or mixed
- $\langle A\rangle = Tr(\rho A)$ irrespective of whether ρ is pure or mixed
- $\langle A\rangle = Tr(\rho A)$ only when ρ is pure state

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\langle A\rangle = Tr(\rho A)$ irrespective of whether ρ is pure or mixed

8) The distinguishing feature of the Bloch vector representing pure versus mixed state is *1 point*

- that both are always equal to 1
- that the pure state is always equal to 1 while the mixed state is more than 1
- that both are always less than 1
- that the mixed state is less than 1 while the pure state is always equal to 1

No, the answer is incorrect.
Score: 0
Accepted Answers:
that the mixed state is less than 1 while the pure state is always equal to 1

9) The density matrix (ρ) of a two-level system $\{|0\rangle, |1\rangle\}$ is given by: $\rho = \frac{1}{5}\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$. This implies: *1 point*

- The probability that a measurement of the system will find it in state |01> is $\frac{1}{5}$
- ρ is a pure state that on measurement can be found in state |00> with probability $\frac{4}{5}$
- ρ is a mixed state
- ρ is a pure state with a probability of $\frac{1}{5}$ that a measurement of the system will find it in state |0> and a probability of $\frac{4}{5}$ that it will be in state |1>

No, the answer is incorrect.
Score: 0
Accepted Answers:
ρ is a pure state with a probability of $\frac{1}{5}$ that a measurement of the system will find it in state |0> and a probability of $\frac{4}{5}$ that it will be in state |1>

10) . A density matrix of the form $\rho = \frac{1}{4}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$ can be written equivalently as: *1 point*

- $\frac{1}{2}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \sqrt{3}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right]$ or $\frac{1}{2}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \sqrt{3}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]$
- $\frac{1}{2}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \sqrt{3}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right]$ or $\frac{1}{2}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\otimes\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \sqrt{3}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]$
- $\frac{1}{2}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sqrt{3}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\otimes\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]$ or $\frac{1}{2}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\otimes\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \sqrt{3}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]$
- $\frac{1}{4}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \sqrt{3}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right]$ or $\frac{1}{4}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\otimes\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \sqrt{3}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{1}{2}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sqrt{3}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\otimes\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]$ or $\frac{1}{2}\left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\otimes\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \sqrt{3}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\otimes\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]$