MATHEMATICS FOR CHEMISTRY

ASSIGNMENT 8 - SOLUTIONS

$\boxed{1}$ $$y'' + \frac{y'}{x} + \frac{y}{x^2} = 0$$

we can apply ~~Frob~~ Frobenius method since $\frac{1}{x}$ does not go to ∞ faster than $\frac{1}{x}$ & $\frac{1}{x^2}$ does not go to ∞ faster than $\frac{1}{x^2}$.

$\boxed{2}$ $$y = \sum_{n=0}^{\infty} c_n x^{n+r} \qquad y' = \quad c_n x^{n+r-1} \qquad y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1) \quad c_n x^{n+r-2}$$

$$x^2 y'' = \sum_{n=0}^{\infty} c_n (n+r)(n+r-1) x^{n+r} \qquad xy' = \sum_{n=0}^{\infty} (n+r) c_n x^{n+r}$$

$$r(r-1) + r \quad 0 \qquad r^2 = 0$$

$\boxed{3}$ $x = 1 \Rightarrow 1 - x^2 = 0$ Singular point.

Clearly, it is a regular SP.

So it can be solved by Frobenius method with $r \neq 0$.

$\boxed{4}$ $$r(r-1) + r - \frac{9}{4} = 0 \Rightarrow r^2 - 9/4 = 0$$

$\boxed{5}$ $x = 1$ is not a singular point, so power series method can be used as $y = \sum_{n=0}^{\infty} a_n (x-1)^n$ i.e. $r = 0$.

$\boxed{6}$ Compare with Bessel Equation

$$x^2 y'' + xy' + (x^2 - v^2) y = 0$$

$\Rightarrow v^2 = 16 \quad v = 4.$

One solution involves $J_4(x)$.

$\boxed{7}$ $$R_{n,l}(r) \propto r^l \mathcal{L}_{n,l}(r) e^{-r/a_0 n}$$

$\boxed{\text{m has to be 0, not 1}}$ since $l = 0$. Degree of polynomial $= n - l - 1$

$= 2$

$\boxed{8}$ $\sum_{n=0}^{\infty} a_n x^{n+r}(n+r)(n+r-1) + \sum a_n x^{n+r}(n+r)$

$+ \sum_{n=0}^{\infty} a_n x^{n+r+2} - \sum_{n=0}^{\infty} 4a_n x^{n+r} = 0$

Coeffiant of x^r : ~~$A=1$~~ $r(r-1)+r - 4 = 0$

$r^2 = 4 \quad r = \pm 2$

If ~~$r=4$~~ Coeffiant of x^{n+r}

$(n+r)(n+r-1)a_n + a_{n-2} - 4a_n = 0$

$$a_n = \frac{-a_{n-2}}{(n+r)^2 - 4} = \frac{-a_{n-2}}{n^2+2nr}$$

Consider n - even

$$a_n = -\frac{a_{n-2}}{n(n+2r)} = \frac{(-1)^2 a_{n-4}}{n(n+2r)(n-2)(n-2+2r)}$$

$$a_n = \frac{(-1)^{n/2} a_0}{n(n-2)(n-4)\dots 2 \cdot (n+2r)(n+2r-2)\dots(2+2r)}$$

If $r=2$

$$a_n = \frac{(-1)^{n/2} a_0}{6^2 \cdot 8^2 \cdot 10^2 \dots n^2 \cdot (n+2)(n+4) \cdot 2 \cdot 4}$$

or $$a_{2n} = \frac{(-1)^n a_0}{2^{2n}(n!)^2} \frac{4 \cdot 2}{(n+2)(n+4)}$$

If $r=-2$ $$a_{2n} = \frac{(-1)^n a_0 \, 2}{2^{2n}(n!)^2}$$

$\boxed{9}$ Solution involves $J_0(r/2)$

$\boxed{10}$ ~~$n=4, l=2$~~ $R_{n,l} \sim r^l$ ~~R~~ Polynomial of degree $n-l-1$.

Ans. None of above