Mathematics for Chemistry - Assignment 7

SOLUTIONS

$\boxed{1}$ $$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + y = 0$$

$$\sum_n n(n-1)a_n x^{n-2} - 2na_n x^n + a_n x^n = 0$$

Coefficent of x^n : $(n+1)(n+2)a_{n+2} - 2na_n + a_n = 0$

$$a_{n+2} = a_n \frac{2n-1}{(n+1)(n+2)}$$

$\boxed{2}$ $$\sum_n n(n-1)a_n x^{n-2} - 2a_n x^n + a_n x^n = 0$$

$$(2+n)(n+1)a_{n+2} - 2a_n + a_n = 0$$

$$a_{n+2} = \frac{a_n}{(n+1)(n+2)}$$

* Does not terminate !!

$\boxed{3}$ $$\sum_n n(n-1)a_n x^{n-2} - 2na_n x^n + a_n x^n = 0$$

$$(n+2)(n+1)a_{n+2} - (2n-1)a_n = 0$$

$$a_{n+2} = \frac{(2n-1)a_n}{(n+1)(n+2)}$$

$$a_4 = \frac{3a_2}{4.3} = \frac{a_2}{4} = \frac{-1a_0}{2\times 4} = -\frac{a_0}{8}$$

$\boxed{4}$ $$P_n(x) = \frac{(-1)^n}{2^n n!}\frac{d^n}{dx^n}(1-x^2)^n$$

$$P_5(x) = \frac{(-1)^n}{2^n n!}\frac{d^5}{dx^5}(1-x^2)^5$$

$$= \frac{-1}{32\times 120}\frac{d^5}{dx^5}\left[-x^{10} + \quad + 5x^8 - 10x^6\right]$$

$$= \frac{-1}{32\times 120}\left[-10\times 9\times 8\times 7\times 6\, x^5 + 5\times 8\times 7\times 6\times 5\times 4\, x^3 - 10\times 6\times 5\times 4\times 3\times 2\, x\right]$$

$$= \frac{1}{8}\left[63x^5 - 70x^3 + 15x\right]$$

$\boxed{5}$ $P_6(x)$ contains x^6, x^4, x^2, x^0

or 6, 4, 2, 0 powers of x.

$\boxed{6}$ $6 = 2 \times 3$

Solution $= a_0 P_2(x) + a_1 S_{odd}(x)$

$\boxed{7}$ $|\vec{L}| = \hbar\sqrt{\ell(\ell+1)}$

$= \hbar\sqrt{6}$

$\boxed{8}$ $\int_{-1}^{1} P_3(x)\, x P_5(x)\, dx = 0$

because $xP_5 = a P_4 + b P_6$

and is independent of $P_3(x)$

$\boxed{9}$ $\int_{-\infty}^{+\infty} \underbrace{H_4(x)\, x H_5(x)} \, dx$ Notice, No e^{-x^2} factor

Involves even polynomials

Integral diverges and goes to infinity.

$\boxed{10}$ Solution is $H_4(x) = 4x^4 - 12x^2 + 3$