Mathematics for Chemistry: Assignment 7

June 15, 2017

1. Consider the ODE
$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + y = 0$$
Solving this ODE using the power series method with $y = \sum_{n=0}^{\infty} a_n x^n$, we get the recursion relation

(a)
$$a_{n+2} = a_n \frac{2n+1}{(n+1)(n+2)}$$

(b)
$$a_{n+2} = a_n \frac{2n-1}{(n+1)(n+2)}$$

(c)
$$a_{n+2} = a_n \frac{n}{(n+1)(n-2)}$$

(d) None of the above

Answer (b)

2. Consider the ODE
$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + y = 0$$
The condition for a solution to this ODE to be a polynomial of order k is
(a) $k = 1$ (b) $k = 2$
(c) $k =$ odd positive integer (d) No positive integer solution for k
Answer (d)

3. Consider the ODE
$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + y = 0.$$
This is solved using the power series method by substituting
$$y = \sum_{n=0}^{\infty} a_n x^n$$
The relation between a_4 and a_0 is
(a) $a_4 = 1/4a_0$ (b) $a_4 = 1/8a_0$ (c) $a_4 = -1/8a_0$
(d) None of the above
Answer (c)

4. Using the Rodrigues formula for Legendre Polynomials
$$P_n(x) = \frac{(-1)^n}{2^n n!}\frac{d^n}{dx^n}(1-x^2)^n$$
the value of the Legendre Polynomial $P_5(x)$ is

(a)
$$\frac{1}{4}\left(42x^5 - 16x^3 + 31x\right)$$

(b)
$$\frac{1}{4}\left(14x^5 - 41x^3 + 30x\right)$$

(c)
$$\frac{1}{4}\left(63x^5 - 70x^3 + 15x\right)$$

(d) None of the above

Answer (c)

5. The powers of x that appear in the expression for the Legendre Polynomial $P_6(x)$ are
(a) 0, 2, 4, 6, 8
(b) 0, 2, 4, 8
(c) 2, 4, 6
(d) None of the above
Answer (b)

6. Let a_0 and a_1 be arbitrary constants. Let $S_{odd}(x)$ and $S_{even}(x)$ denote infinite series in x with only odd and even powers respectively. The general solution of the DE
$$(1-x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 6y = 0$$
can be expressed in the form

(a)
$$a_0 P_2(x) + a_1 S_{odd}(x)$$

(b)
$$a_0 S_{even}(x) + a_1 P_1(x)$$

(c)
$$a_0 S_{even}(x) + a_1 P_3(x)$$

(d) None of the above

Answer (a)

7. The angular momentum ($|\vec{L}|$) of a rigid rotor with $l = 2, m = 1$ is equal to
(a) 6 $\hbar$ (b) 6 $\hbar^2$ (c) $\sqrt{6}\hbar$ (d) None of the above
Answer (c)

8. The value of
$$\int_{-1}^{+1} P_3(x) x P_5(x) dx$$
is equal to
(a) 0 (b) 6 (c) 2 (d) $\sqrt{2}/3$
Answer (a)

9. The value of
$$\int_{-\infty}^{+\infty} H_4(x) x H_5(x) dx$$
is equal to
(a) 0 (b) $1920\sqrt{\pi}$ (c) $496\sqrt{\pi}$ (d) None of the above
Answer (d)

10. The polynomial below that solves

$$\frac{d^2y}{dx^2} - 2x\frac{dx}{dy} + 8y = 0$$

is
(a) x (b) $x^4 - 3x^2 + 2$ (c) $4x^4 - 12x^2 + 3$
(d) None of the above
Answer (c)