# Mathematics for Chemistry - Assignment 6 - Solutions

$\boxed{1}$ $\begin{bmatrix} dx_1/dt \\ dx_2/dt \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Eigenvalues $\begin{vmatrix} 3-\lambda & 2 \\ 2 & 3-\lambda \end{vmatrix} = 0$ $\quad \lambda = 3 \pm 2 \quad \lambda = 1$ or $\lambda = 5$

$\lambda = 1$ $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_1 = -x_2$. Choosing $x_2 = 1$, we get $x_1 = -1$

Eigenvector $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$. Similarly $\lambda = 5$ gives $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Solution $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{t} + B \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{5t}$

$\boxed{2}$ $y'' + 6y' + 9y = 0 \qquad y = e^{\lambda x}$

gives $\lambda^2 + 6\lambda + 9 = 0 \Rightarrow (\lambda+3)^2 = 0 \qquad \lambda = -3$

General solution: $y = e^{-3x}[A + Bx]$

$\boxed{3}$ $y'' + 4y' + 9y = 0 \qquad \lambda^2 + 4\lambda + 9 = 0$

$$\lambda = \frac{-4 \pm \sqrt{16-36}}{2} = -2 \pm \sqrt{5}$$

$y = e^{-2x}\left[Ae^{i\sqrt{5}x} + Be^{-i\sqrt{5}x}\right]$

$y(0) = 0 \Rightarrow A + B = 0 \qquad A = -B$

$y'(0) = A(-2 + i\sqrt{5}) - A(-2 - i\sqrt{5})$

$= 2A\sqrt{5}\, i = \sqrt{5} \Rightarrow A = 1/2i$

$$y = e^{-2x}\frac{\left[e^{i\sqrt{5}x} - e^{-i\sqrt{5}x}\right]}{2i} = e^{-2x}\sin\sqrt{5}\,x$$

$\boxed{4}$ $y = x$ clearly satisfies the DE.

$\boxed{5}$ $y'' + 16y = 4x^2$

$y = A\sin 4x + B\cos 4x + y_p(x)$

$= A\sin 4x + B\cos 4x + C_0 + C_1 x + C_2 x^2 \Rightarrow C_2 = 1/4,\ C_0 = -1/32$

$C_1 = 0$

Ans. (C)

$\boxed{6}$ $y'' + y = \sin 2x$

Clearly $A\sin x + B\cos x - \frac{1}{3}\sin 2x$ satisfies

$\boxed{7}$ $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = \sin(\omega t)$

$x = e^{kt} \Rightarrow (k^2 + 2k + 5)e^{kt}$

$\omega = \sqrt{5}/_1$ for resonance

$\boxed{8}$ $x^2y'' + 14xy' + 12y = 0$

Solution: $x^m$

$m^2x^m - mx^m + 14mx^m + 12x^m = 0$

$\Rightarrow m^2 + 13x^m + 12 = 0$

$(m+12)(m+1) = 0 \qquad m = -12, -1.$

$y = Ax^{-12} + Bx^{-1}$

$\boxed{9}$ $x^2y'' + 3xy' + 2y = 0$

Solution $x^m$: $m(m-1) + 3m + 2 = 0$

$m^2 + 2m + 2 = 0 \qquad m = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$

Solution $y = Ax^{-1+i} + Bx^{-1-i}$ (None of above)

$\boxed{10}$ $y'' + \frac{4y'}{x} + \frac{6y}{x^2} = 1$

$x^2y'' + 4xy + 6y = x^2$

Solution to homogeneous equation

$y_h = x^m \Rightarrow m^2 + 3m + 6 = 0$

$m = \frac{-3 \pm \sqrt{3}i}{2}$

Solution: None of