Mathematics for Chemistry: Assignment 6

June 15, 2017

1. Consider the system of coupled first order equations:

$$\begin{aligned} \frac{\mathrm{d}x_1}{\mathrm{d}t} &= 3x_1 + 2x_2 \\ \frac{\mathrm{d}x_2}{\mathrm{d}t} &= 2x_1 + 3x_2 \end{aligned}$$

has a general solution of the form (where A and B are arbitrary constants)

(a)
$$A\begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t} + B\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-5t}$$

(b)
$$A\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + B\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-5t}$$

(c)
$$A\begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t} + B\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{5t}$$

(d)
$$A\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{3+it} + B\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{3-it}$$

Answer (c)

2. Consider the second order differential equation

$$y'' + 6y' + 9y = 0$$

The general solution of this equation is (where A and B are arbitrary constants)
(a) $Ae^{3x} + Be^{-3x}$ (b) $Ae^{4x} + Be^{2x}$ (c) $Ae^{3x+2ix} + Be^{3x-2ix}$(d) $Ae^{-3x} + Bxe^{-3x}$

Answer (d)

3. Consider the second order differential equation

$$y'' + 4y' + 9y = 0$$

with the boundary conditions $y(0) = 0$, and $y'(0) = \sqrt{5}$. The particular solution of this ODE is

(a) $e^{2x}\sin(5x)$ (b)$e^{2x}\cos(5x)$ (c)$e^{-2x}\cos(5x)$ (d) $e^{-2x}\sin(\sqrt{5}x)$

Answer(d)

4. The ODE
$$y'' - \frac{x\sin(x)}{\sin(x) - x\cos(x)}y' + \frac{\sin(x)}{\sin(x) - x\cos(x)}y = 0$$
has $\sin(x)$ as one of the solutions. The other linearly independent solution is
(a) x (b) $\cos(x)$ (c) $\sin(2x)$ (d) $\sin(x) - x\cos(x)$

Answer(a)

5. The general solution of the second order nonhomogeneous equation
$$y'' + 16y = 4x^2$$
is (A and B are arbitrary constants)

(a)
$$y = A\sin(4x) + B\cos(4x) + \sin(4x)\cos(4x)$$

(b)
$$y = A\sin(4x) + B\cos(4x) + \ln(\frac{1+\sin(4x)}{\cos(4x)})$$

(c)
$$y = A\sin(4x) + B\cos(4x) + (\frac{-1}{32} + \frac{x^2}{4})$$

(d) None of the above

Answer (c)

6. The second order ODE $y'' + y = \sin 2x$ has a general solution of the form (where A and B are arbitrary constants)

(a)
$$A\sin x + B\cos x + \sin 2x$$

(b)
$$A\sin x + B\cos x - \sin 2x$$

(c)
$$A\sin x + B\cos x - \frac{1}{3}\sin 2x$$

(d)
$$A\sin x + B\cos x + 1$$

Answer (c)

7. A forced damped harmonic oscillator satisfies the differential equation
$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + 5x = \sin(\omega t)$$
The condition for resonance in this oscillator is
(a) $\omega = 5$ (b) $\omega = \sqrt{5}$ (c) $\omega = 2$ (d) $\omega = \sqrt{2}$

Answer (b)

8. The solution of the equation
$$x^2y'' + 14xy' + 12y = 0$$
is (where A and B are arbitrary constants) (a) $Ax^12 + Bx$ (b) $Ax^{-12} + Bx^{-1}$ (c) $Ax^{7+i} + Bx^{7-i}$ (d) None of the above

Answer (b)

9. The solution of the equation

$$x^2y'' + 3xy' + 2y = 0$$

is (where A and B are arbitrary constants)

(a) $Ax^{-1} + Bx$ (b) $Ax^{-i} + Bx^i$ (c) $Ax^{1+i} + Bx^{1-i}$ (d) None of the above

Answer (d)

10. The general solution of the differential equation

$$y'' + \frac{4y'}{x} + \frac{3y}{x^2} = 1$$

is (where A and B are arbitrary constants)

(a)

$$y = \frac{A}{x^2} + \frac{B\log(x)}{x^2} + \frac{x^2}{16}$$

(b)

$$y = Ae^x + Be^{-x} + 2x + 4x^2 - 1$$

(c)

$$y = Ax + \frac{B}{x^2} + \log(x) + 4x^2 - 1$$

(d) None of the above

Answer (d)