Mathematics for Chemistry - Assignment 5 - Solutions

$\boxed{1}$ Nonhomogeneous nonlinear ODE due to y^2 term and xy term

$\boxed{2}$ Linear nonhomogeneous ODE

$\boxed{3}$ Homogeneous linear ODEs.

$\boxed{4}$ Nonhomogeneous nonlinear ODEs.

$\boxed{5}$ 3 arbitrary constants in the general solution of 3rd order ODE and no arbitrary constants in particular solution of 1st order ODE.

$\boxed{6}$ $\frac{dy}{y} = -3x \Rightarrow \ln y = -\frac{3x^2}{2} + c$

Put $x=0$, $y=1 \Rightarrow c=0$ $\quad y = e^{-\frac{3x^2}{2}}$

$\boxed{7}$ $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1 - (1-4y) = 4y$

Clearly $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ is not a function only of x

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No integrating factor that depends only on x or only on y.

$\boxed{8}$ $\frac{\partial M}{\partial y} = 2 = \frac{\partial N}{\partial x} \Rightarrow$ Exact differential

$du = (2y+3x)\,dx + (2x+y)\,dy = 0$

$\frac{\partial u}{\partial x} = 2y+3x \Rightarrow u = 2xy + \frac{3x^2}{2} + f(y)$

$\frac{\partial u}{\partial y} = 2x + y \Rightarrow 2x+y = 2x + \cancel{3x} + \frac{\partial f}{\partial y} \Rightarrow f = \frac{y^2}{2}$

Solution : $2xy + \frac{3x^2}{2} + \frac{y^2}{2} = C$ OR $4xy + 3x^2 + y^2 = c$

$\boxed{9}$ $y' + y = \sin x$

$y_h = Ae^{-x}$ $y_p = C\sin x + D\cos x$

Substituting $y = y_h + y_p$, we get

$C\cos x - D\sin x + C\sin x + D\cos x = \sin x$

Comparing $\cos x$ terms

$C + D = 0$ $C = -D$

Comparing $\sin x$ terms

$C - D = 1$ $D = -1/2$ $C = 1/2$

$y = Ae^{-x} + \frac{1}{2}(\sin x - \cos x)$

$\boxed{10}$ $y' + 2y = 3e^{-2x}$ $y_h = Ae^{-2x}$ $y_p = Bxe^{-2x}$

Substituting gives

$Be^{-2x} - 2Bxe^{-2x} + 2Bxe^{-2x} = 3e^{-2x}$

$\Rightarrow B = 3$

$y = Ae^{-2x} + 3xe^{-2x}$