

Mathematics for Chemistry - Assignment 4 Solution

1 Det $\begin{vmatrix} a & -1 & 0 & 0 \\ 1 & a & 1 & 0 \\ 0 & 1 & a & 1 \\ 0 & 0 & 1 & a \end{vmatrix} = 0 \Rightarrow a^4 - a^2 - 1 = 0$
 $a^4 = a^2 + 1$

2 $M = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & -6 & -2 \\ 4 & 21 & 10 \end{bmatrix} \xrightarrow{\substack{R_2 - \frac{4}{3}R_1 \\ R_3 - \frac{1}{3}R_1 \\ R_4 - \frac{4}{3}R_1}} \begin{bmatrix} 3 & 2 & 1 \\ 0 & -5/3 & 5/3 \\ 0 & -20/3 & -7/3 \\ 0 & 55/3 & 26/3 \end{bmatrix} \xrightarrow{\substack{R_3 - 4R_2 \\ R_4 + 11R_2}} \begin{bmatrix} 3 & 2 & 1 \\ 0 & -5/3 & 5/3 \\ 0 & 0 & -9 \\ 0 & 0 & 27 \end{bmatrix}$

Rank $M = 3$

$\tilde{M} = \begin{bmatrix} 3 & 2 & 1 & 7 \\ 4 & 1 & 3 & 11 \\ 1 & -6 & -2 & 0 \\ 4 & 21 & 10 & 18 \end{bmatrix} \xrightarrow{\substack{R_2 - \frac{4}{3}R_1 \\ R_3 - \frac{1}{3}R_1 \\ R_4 - \frac{4}{3}R_1}} \begin{bmatrix} 3 & 2 & 1 & 7 \\ 0 & -5/3 & 5/3 & 5/3 \\ 0 & -20/3 & -7/3 & -7/3 \\ 0 & 55/3 & 26/3 & 26/3 \end{bmatrix} \xrightarrow{\substack{R_3 - 4R_2 \\ R_4 + 11R_2}} \begin{bmatrix} 3 & 2 & 1 & 7 \\ 0 & -5/3 & 5/3 & 5/3 \\ 0 & 0 & -9 & -9 \\ 0 & 0 & 27 & 27 \end{bmatrix}$

Rank $\tilde{M} = 3$

Equations are sufficient and consistent.

3 Determinant = $1 - 4b^2$

4 Eigenvalues are $1, 1, 1 - 2b, 1 + 2b$ ← Largest eigenvalue = $1 - 2b$ since $b < 0$

5 Hermitian matrices, Symmetric is used for real matrices only. So symmetric real matrices are also Hermitian and have real eigenvalues.

7 Calculate inverse explicitly to get (c)

6 Clearly not symmetric. Also orthogonal.

8 No solution. Eq 1 - E2 $\Rightarrow 3x + 7y - z = -6$

9 $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} -1.87 \\ -1.23 \\ 5 \end{bmatrix}$

10 By procedure as above, we get $(0, -0.29, -1.71)$